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Analysis of portfolio optimization with inequality constraints

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Analysis of portfolio optimization with inequality constraints

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Abstract. In investing, generally an investor wants an optimum portfolio. This means that the forming portfolio has minimum risk, maximum return or can also be a combination of both with other constraints determined by the investor. These constraints could be in the form of short selling, amount of owning funds, return target, risk target or other constraints. Short selling constraints are inequality in the mathematical model, while other constraints can be in the form of equalities or inequalities. This paper discusses the portfolio optimization with these inequality constraints. In addition, we will provide an example for this portfolio optimization application by analysing portfolio that consists of shares in the LQ45 index. We do this analysis with Solver that is available in Microsoft Excel.

1. Introduction

To maintain the values of money due to inflation, an investor generally invests in financial instruments such as bonds, stocks, deposits, and others. Thus investors need to form a portfolio so that the return generated can exceed inflation that occurs. The portfolio itself can be interpreted as a collection of investment assets, such as bonds, shares, property, gold, and other financial instruments. The stock portfolio means a collection of investment assets in the form of shares, both owned by individuals and companies. Portfolio optimization is a process of selecting the proportion of various assets in a portfolio that makes the portfolio better than others based on several criteria, such as: minimum risk and / or maximum return with some constraints given. In this study, we only limited the stock portfolio model.

In previous research [3], we have discussed about minimizing risk and minimizing risk with certain return targets accompanied by the condition of no short-selling. Then, we applied these models to a portfolio consisting of stocks included in the LQ45 index with various scenarios. The model contained in [3] is solved by utilizing the penalty function. This model certainly has its drawbacks, namely the result of the decision variables is still in the form of a proportion of funds. Whereas, stocks are purchased in lots (note that 1 lot is 100 shares). For this reason, we refine that model so that the result of the decision variables is the number of stock lots that need to be purchased in order to get the optimum portfolios [4].

In [4], besides the no short-selling constraint, other constraints in the portfolio optimization models are in the form of equalities. One of them is the allocation of funds. Funds owned by investors are used entirely in the formation of an optimum portfolio. The investor does not need to use all his funds to form an optimum portfolio. An investor can just use some of the available funds in forming an optimum portfolio. Also in [4], the return target must be achieved in the formation of the optimum portfolio. However, investors can just give a minimum limit of the desired return target. Of course, the bigger the target is not a problem for them. In addition, investor can also limit the maximum risk that can be tolerated by him. These constraints are inequality in the mathematical model. The models with constraints in the form of inequality will be discussed in this paper. For example, we carry out portfolio analysis consisting of LQ45 index shares.

At present, the LQ45 stock index which was launched in February 1997 includes indicators of shares in the capital market in Indonesia. LQ45 uses 45 selected stocks with criteria determined by the Indonesia Stock Exchange, including liquidity and market capitalization. According to [2], the value of transactions in the regular market is the main measure of liquidity, and since January 2005 the number of trading days and transaction frequency has been added as a measure of liquidity. Shares in the LQ45 index will be evaluated every 3 months and the replacement of shares into the LQ45 index is conducted every six months, namely in the beginning of February and August. The stocks that are listed in the LQ45 index can be found in [5].

In addition to the constraints previously mentioned, as well as [4], we also determine the number of lot shares to be purchased in the formation of an optimum portfolio. This is consistent with the regulation in Indonesia where an investor can buy shares in lots, namely 1 lot = 100 shares. The results are obtained with the help of Solver in Microsoft Excel. The history data of stock prices is downloaded from [6] with a span of one year, from March 1, 2017 to February 28, 2018.

2. The Models

Suppose that there are n assets in a portfolio and r_{ij} denotes the percentage return rate from the asset-i in the j-th period with $i = 1,2,\dots,n$ and $j = 1,2,\dots,m$ and assumed $m > n$. Furthermore, let y_i ($i = 1,2,\dots,n$) denotes the proportion of the amount of investment for the asset-i with with $\sum_{i=1}^{n} y_i = 1$. The portfolio risk (V) can be defined as the variance of a portfolio [1], that is

$$V = \boldsymbol{y}^T Q \boldsymbol{y}$$

with

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \text{ and } Q = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

The matrix Q is the variance-covariance matrix of historical stock returns. In [1], Biggs has discussed about portfolio optimization with several criteria, including minimizing risk, maximizing return, minimizing risk with a particular target for return, and maximizing returns with a particular target for risk. Some of these criteria were added to the terms without short-selling. However, all of these models are used to form a portfolio consisting of assets in general. In this paper, we specifically want to discuss the portfolio which only consists of stocks. Therefore, we need to change these models so that the results of the decision variables are the number of lots of stocks that need to be purchased rather than the proportion. We have discussed these models in [4].

In [4], we have discussed the following model.

$$\min \ V = \boldsymbol{y}^T Q \boldsymbol{y}$$

subject to

$$100 \sum_{i=1}^{n} z_i P_i = M$$

$$\frac{100}{M} \sum_{i=1}^{n} \bar{r}_i z_i P_i = R_p$$

$$z_i \in \mathbb{Z}^+$$

where
z_i: the number of lots of stock i,
P_i: the current price of stock i,
M: the amount of funds to be invested

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$\bar{r}_i$: the mean return for stock i,
R_p: the return target desired by the investor.

Because stocks must be purchased in units of lots (1 lot = 100 shares) then we have the relationship between z_i and y_i, that is

$$y_i = 100\frac{z_i P_i}{M}$$

Of course, the investor does not need to use all his funds (M) to achieve his investment objective. Therefore, the constraint of the equality can be changed to inequality, which is smaller or equal to. Similarly, for the return target is changed to be greater or equal to. That is, the investor can expect a minimum portfolio return of R_p. So, we have the following model.

$$\min \boldsymbol{V} = \boldsymbol{y}^T \boldsymbol{Q} \boldsymbol{y} = \frac{\mathbf{10^4}}{\boldsymbol{M}^2}\left(\sum_{i=1}^{n} \boldsymbol{\sigma}_i^2 \boldsymbol{z}_i^2 \boldsymbol{P}_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \boldsymbol{\sigma}_{ij} \boldsymbol{z}_i \boldsymbol{P}_i \boldsymbol{z}_j \boldsymbol{P}_j\right) \tag{1}$$

subject to

$$100\sum_{i=1}^{n} z_i P_i \le M$$
$$\frac{100}{M}\sum_{i=1}^{n} \bar{r}_i z_i P_i \ge R_p$$
$$z_i \in \mathbb{Z}^+$$

The number of lots of stock i, z_i, is a positive integer because short selling is not allowed.

The objective function in (1) and its constraints above is more suitable for conservative investors where the risks that might be faced want to be as small as possible. We called this optimization problem with **Model1**. For an aggressive investor, the person is more concerned with a large return than the smallest possible risk and he limits the risk of portfolio by V_a. So, an aggressive investor can use the following model (we use the model that is discussed in [1] with replacing y_i by z_i).

$$\min \boldsymbol{R} = -\bar{\boldsymbol{r}}^T \boldsymbol{y} = \sum_{i=1}^{n} \bar{\boldsymbol{r}}_i \boldsymbol{y}_i = \frac{\mathbf{100}}{\boldsymbol{M}}\sum_{i=1}^{n} \bar{\boldsymbol{r}}_i \boldsymbol{z}_i \boldsymbol{P}_i \tag{2}$$

subject to

$$100\sum_{i=1}^{n} z_i P_i \le M$$
$$\frac{10^4}{M^2}\left(\sum_{i=1}^{n} \sigma_i^2 z_i^2 P_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \sigma_{ij} z_i P_i z_j P_j\right) \le V_a$$
$$z_i \in \mathbb{Z}^+$$

where $\bar{\boldsymbol{r}} = (\bar{r}_1\ \bar{r}_2\ \cdots\ \bar{r}_n)^T$. As the previous model, short selling is not allowed in this model too, so z_i must be a positive integer. We called this optimization problem with **Model2**. Both models with objective functions in (1) and (2) and their constraints are classified as mixed integer nonlinear programming problem. We use the Solver which is a Microsoft Excel tool to solve both of these models because the Solver is easy to use. The Solver tool uses several algorithms to find optimal solutions. These algorithms was developed by Leon Lasdon, University of Texas, and Alan Waren, Cleveland State University, and enhanced by Frontline Systems, Inc. [7]

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3. The Results

To solve the models in previous section, we need to calculate a variance-covariance matrix Q. The formula is [1]:

$$\sigma_{ii} = \sigma_i = \frac{1}{m}\sum_{j=1}^{m}\left(r_{ij} - \bar{r}_i\right)^2$$

and

$$\sigma_{ij} = \frac{1}{m}\sum_{k=1}^{m}(r_{ik} - \bar{r}_i)\left(r_{jk} - \bar{r}_j\right)$$

with $\bar{r}_i$ representing the average return of the stock i. We use the stocks in the LQ45 and we take the historical closed stocks price data from March 1, 2017 until February 28, 2018 that we downloaded on March 10, 2018 at yahoo finance [6]. There are 8 stocks with the negative average return, so we omit them (we only use 37 stocks in the LQ45 index rather than 45 stocks). The amount of funds to be invested M that we used is Rp 1 billion and we use initial guest for $z_i = 50$ for each i. For **Model1**, we use the minimum return target $R_p = 0.5\%$ whereas for **Model2**, we use the maximum risk $V_a = 0.2\%$. Here are the results :

Table 1. Selected portfolio using Model1

Stock	Price (Rp)	Lots	Stock	Price (Rp)	Lots
BBCA	23,175	43	INDY	4,220	205
BBNI	9,625	14	INKP	11,150	213
BBRI	3,790	33	INTP	21,600	25
BBTN	3,760	287	MEDC	1,500	747
BKSL	228	2891	UNTR	36,525	41
BRPT	2,570	234			
		$V = 0.017\%$			
		$R_p = 0.5\%$			

Table 2. Selected portfolio using Model2

Stock	Price (Rp)	Lot(s)
ANTM	940	1
INDY	4,220	2
INKP	11,150	896
	$V_a = 0.121\%$	
	$R = 0.969\%$	

From Table 1 and Table 2, we can see that the portfolios are only formed from 11 stocks and 3 stocks, respectively. For both **Model1** and **Model2**, an investor makes the biggest investment in INKP. Even, on **Model2**, almost all funds were invested in INKP. Actually, the average return of INKP is the highest among other stocks in the period that we take. So, this result is very reasonable.

4. Conclusions and Further Research

For an aggressive investor, he can invest for almost all his fund in INKP. For a moderate investor, he can select the portfolio of 11 stocks with the smaller risk and return rather than an aggressive investor. For further research, portfolio rebalancing seems to be considered to keep the portfolio set up by investors to provide a return that matches their expectations. In addition, the cost of stock purchase transactions need to be considered, as well.

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