Contributions to the Financial Mathematics of Energy Markets

Ferry Jaya Permana

CONTRIBUTIONS

TO THE FINANCIAL MATHEMATICS

OF ENERGY MARKETS

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OF ENERGY MARKETS

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This thesis is dedicated to
my mother,
my 'family',
and the memory of my father.

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Preface

Commodities surround us everywhere in our daily life. People consume staple foods such as wheat, rice, potato or corn, and also need vegetables, fruits and meats. Cotton is used to make clothes. Metals and wood are essential raw material for making tools, machinery or even cars and houses. Moreover, in the modern world, the need for oil, gas or electricity as energy resources is high and still growing. Commodities have major impact on economies of developed and developing countries. As a result, worldwide commodity markets have grown explosively in the past several decades.

Commodities are defined as consumption assets whose scarcity has a major impact on the world and country-specific economic development (?). In the old days, commodities were traded in the so-called *spot markets* (similar to common market places), where buyers and sellers met to make transactions for immediately delivery. The history of "modern" commodity markets has begun in the 16th century, when growers in Japan sold their rice at the time of planting in order to finance the production. This transaction did the first so-called *forward contract*. Such contracts did become rapidly popular. In the 18th century, numerous forward contracts on potatoes and other agricultural products took place in the USA. Standardization of such contracts in terms of quantity, quality and delivery date of a commodity has become inevitable. This triggered the establishment of NYCE (New York Cotton Exchange) in 1842 and CBOT (Chicago Board of Trade) in 1848.

In the Netherlands, commodity markets began in the 16th century. Forward contracts on tulip bulbs have been traded in the Netherlands since the early 1600's. It was not long before many speculators joined in and led the market to a crash, by failing to honor their agreements. Soon afterwards, such transactions were even declared illegal. However, commodity trade in the Netherlands was not affected by this unfortunate incident. The Amsterdam Exchange (Amsterdamse Beurs) is considered the oldest stock exchange in the world. It started trading in the 16th century, when Dutch traders decided to take charge of the spice import from Asia. The prime trading and shipping companies of Holland merged to form one large company called VOC (Verenigde Oostindische Compagnie). In order to finance their ships and equipment, in 1602 the VOC established the Amsterdam Exchange, which is still active now as part of the European Stock Exchange Euronext.

Of all commodities, energy has become the biggest market-traded commodity (although modern commodity markets have roots in the trading of agricultural products). This happened after the deregulation in oil and natural gas industries in the 1980s, followed by deregulation in electricity industry in the 1990s. Before the deregulation, prices were set by the regulator, i.e., governments. Energy prices were relatively stable, but consumers had to pay high premiums for inefficient cost, e.g., complex cross-subsidies from area with surpluses to area with shortages or inefficient technology. Deregulation led to a free market with more competitive prices, but revealed that energy prices are the most volatile among all commodities, which exposed both energy producers and consumers to many financial risks.

Commodity (and especially energy) prices are much more volatile than prices on stocks, bonds or other financial indices. This high volatility is caused by rapid changes in supply and demand due to many reasons. Fluctuations of oil prices often are associated with fluctuations in supply due to political events, e.g., the Gulf war or the terrorist attacks of September 11, 2001. For natural gas and electricity, price fluctuations are mostly led by weather conditions, i.e., fluctuations in demand.

As a consequence of the extreme volatility of energy prices, energy market participants are susceptible to significant market risk (i.e., the risk associated with the uncertainty about the price). There are several ways to manage the price risk. The simplest way is to buy and store the physical commodity (e.g., the oil) when prices fall and use it when prices increase. However, this is an expensive way of managing the price risk because the storage costs are usually quite high. And for electricity this does not work at all since it is completely non-storable and must be consumed once generated.

A cheaper way of managing the price risk is using so-called *derivatives contracts*, or simply *derivatives*. The simplest derivatives contract is the forward contract we already mentioned, where the purchase (and hence the price) of a commodity is agreed now, but the commodity itself is delivered sometime in the future. A futures contract is very similar to a forward contract, only it is standardized (in terms of quality and quantity of the underlying commodity) and it is traded via an exchange and not directly between two counterparties.

There are myriads of other derivatives in modern financial and commodity markets. One thing they have in common is that they cannot exist without the underlying asset (e.g., a commodity) and their price is derived from the price of that underlying asset. Derivatives have become very popular in the past few decades, as they can be a very effective and efficient tool for managing risk, if used wisely. At present, almost all activity in commodity markets takes place in the trading of commodity derivatives (especially forward and futures contracts) and not commodities themselves. For example, in the case of oil, trading volumes in derivatives markets are nine times larger than those occurring in trading of actual (physical) oil, and this ratio is consistently increasing with the arrival of new financial players.

Derivatives are essentially bets on the prices of the underlying asset. And as one can bet on the direction of a stock or oil price, one can also bet on e.g., what kind of weather it will be this winter in The Netherlands, or whether a certain catastrophic event will take place (e.g., whether a strong hurricane hits Texas this autumn). The realization of this fact has led to the emergence of two new classes of derivatives: weather and catastrophe derivatives.

Weather plays an important role in many industries: in agriculture, weather affects crop harvests. The energy industry is also very susceptible to weather: production of electricity (especially hydro-electricity) heavily depends on the rainfall. Moreover, severe weather conditions can raise the demand for natural gas and electricity. Tourism, construction and many other industries are also susceptible to risks associated with weather conditions. Nowadays many of these industries can protect their profits from weather risks by buying or selling appropriate weather derivatives. Such derivatives are traded in the US, Europe and Japan, on many exchanges such as Chicago Mercantile Exchange.

Catastrophe derivatives bear more similarities to insurance policies: a holder of a catastrophe derivative will receive a substantial payment if a certain catastrophic event takes place (of course, this holder has to pay a price for such a derivative at the start, just as one has to pay an insurance premium). The big difference with insurance, however, is that catastrophe derivatives are not issued by insurance companies (although they use such derivatives very actively now in their daily operations), but are freely traded on exchanges. It is clear who would buy a catastrophe derivative: a construction company, a real estate developer or an insurance company which will have to pay large compensations in case of a catastrophic event. But who would sell such a derivative? Anyone who wants to diversify their portfolio, since occurrence of a catastrophic event (and hence, a cash flow associated with a catastrophe derivative) is almost uncorrelated to movements of financial markets or financial crises. In this respect, catastrophe derivatives are the so-called *zero-beta assets*, i.e., their prices do not move together with prices of stocks or financial indices.

Another interesting new development in commodity trading has been the introduction of the so-called *emission trading*, or emission markets. On these markets, the allowances to companies to emit greenhouse gasses such as CO_2, are freely traded as any other commodity. Since the ratification of the Kyoto Protocol in 2006, each participating countrys government sets a limit on the amount of carbon emission that any company can produce. If a company pollutes more than its limit, it has to buy extra emission allowances from a company that pollutes less than its limit. In this way, carbon emission allowances have become a traded commodity. These allowances and derivatives on them are traded on many exchanges worldwide, e.g., on the European Energy Exchange.

The simplest commodity derivatives are forward and futures contracts. Another popular derivatives contract is an option: a right (and not an obligation, as it is the case with forward or futures contract) to buy or sell a certain asset (e.g., a commodity) at a fixed price on a fixed date. This is the so-called plain vanilla, or European option. However, real financial markets (and especially commodity markets) are more like zoos, populated by all kinds of derivatives, which can be much more complicated agreements. Such complicated contracts are called exotic derivatives, and they fascinate finance researchers, just as exotic birds or animals fascinate most people.

Energy options and volatility

The first option trading was legitimized in 1934 in the USA for some agricultural products. For the next several decades, the option trading was virtually inexistent. The main reason for this was the fact that it was not clear what a fair price of an option should be and how the seller of an option contract can manage risks associated with it. Then, in 1973, American economists Fischer Black and Myron S. Scholes published a groundbreaking article, which at once answered all these questions and introduced the famous Black-Scholes formula for the fair price of an option. Also in 1973, another economist Robert C. Merton published a paper expanding the mathematical understanding of the options pricing model of Black and Scholes. In the same year, the Chicago Board Options Exchange was established and began trading standardized call options. The impact of the work by Black, Scholes and Merton was so enormous, that trading in options grew exponentially since then and reached the total volume of 20 trillion US dollars worldwide in 1995 (from practically zero in 1972). In recognition of their groundbreaking work, Merton and Scholes were awarded the 1997 Nobel Prize in Economics for the famous Black-Scholes option price formula; Fisher Black unfortunately was ineligible, having died in 1995. Since then, the option trading has been dramatically growing even further, not only in traded volumes (in 2006 the option trading volume has already reached 270 trillion US dollars), but also in the types of traded options. Options have become a practically standard tool of risk management.

On a more academic note, the work by Black, Scholes and Merton gave rise to an entire new discipline on the border between mathematics and finance, the so-called *financial mathematics* or *financial engineering*. It has grown rapidly as a new scientific area, has given a lot of inspiration for researchers and provided many new contributions in both mathematics and finance. The present thesis is also an attempt to contribute to the area of mathematical finance.

There are two classes of derivative pricing models: analytical and numerical approaches. Analytical approaches are particularly attractive for practitioners: they usually provide a closed form expression for a derivative's price, so it is fast and easy to implement, while numerical approaches are more flexible, but tend to be time consuming.

Most commodity derivatives are exotic, i.e., more complicated than European options. There are inherent reasons for this. For example, most energy delivery contracts are based on the average oil price over a certain period of time. In this case, a so-called *Asian option* is better suited for risk management purposes: for Asian options, the settlement price depends on the average asset price over a certain time period and not on the asset price on one specific date, as it is the case with European options. Another example arises from the fact that oil is physically traded twice: as a refinery feedstock and as a refinery product. So the portfolio of most energy companies is a basket of several assets For example, a refinery company must buy crude oil and sell refinery products (e.g., gasoline and heating oil). Hence, its portfolio consists of a so-called short (i.e., negative) position on crude oil and long positions on gasoline and heating oil. A power station is a similar example: it must buy fuel (e.g., natural gas) to produce and sell electricity. A so-called *basket option,* an option whose payoff depends on the weighted sum of the assets in a portfolio, is an effective risk management tool in this case.

The Black-Scholes model heavily depends on the assumption that the price process of the asset underlying an option is a Geometric Brownian Motion (GBM). It means that the underlying asset (e.g., stocks, futures) price's distribution is log-normal and the price log-returns or relative returns are normally distributed. Unfortunately, the celebrated Black-Scholes formula cannot be directly applied for valuing and managing most exotic options. For example, in cases of Asian or basket options, Black-Scholes approach fails due to the fact that the arithmetic average or the weighted sum of log-normal random variables is not log-normal anymore. Valuation of exotic derivatives (such as Asian or basket options) requires specific tools, usually much more complex than the Black-Scholes formula.

Our main contribution in the area of exotic derivatives is a new method for valuing and hedging general basket options, i.e., options on portfolios containing several assets and also long/short positions. This method uses a generalized of family log-normal distributions, the so-called *GLN (Generalized log-normal) distribution,* to approximate the basket distribution. It can deal with negative skewness and negative values, typical for general baskets. The GLN approach leads to closed formulae for option's price and greeks, it allows us to stay within Black-Scholes framework, and can easily be extended to Asian basket options.

Asset price volatility is the main parameter determining the price of an op-