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# A MATHEMATICAL MODEL FOR ELECTION TIMING

Dharma Lesmono[1]
[1]Department of Mathematics, Parahyangan Catholic University
Jalan Ciumbuleuit 94 Bandung 40141
West Java , Indonesia
E-mail: jdharma@home.unpar.ac.id

**Abstract.** We consider a mathematical model for election timing in a Majoritarian Parliamentary System where the government maintains a constitutional right to call an election. This model is based on the two-party-preferred data (poll data) that measure the popularity of the government and the opposition in Australia over a long period of time. A term structured volatility model is proposed to describe the dynamic of those poll data. In addition to the constitutional right, it is assumed that the government can use some control tools termed as `boosts' to induce shocks to the opinion polls by making timely policy announcements or economic actions. These `boosts' lift the government's popularity and their effects upon the early-election exercise boundary are studied.

**Key words**: Optimal stopping; dynamic programming, election timing.

## 1. Introduction

Election timing in a Majoritarian Parliamentary System is a crucial decision for the government in order to stay longer in power. By announcing the election at the right time, the government can maximize its expected remaining life in power. This condition can be found in many countries such as Australia, New Zealand, Canada, and the United Kingdom. There are many indicators considered by the government before coming up with the decision to call an early election such as the economic growth, inflation rate, unemployment level and any other factors in politics and stability within the country.

Some authors ([4], [7], [9], [12], [13], [15]) have used forecasting techniques to predict election outcomes. They have used predictor variables including opinion polls. In particular, Brown and Chappell Jr. [4] observed US Presidential Elections 1952-1992 and used regression analysis to develop a model that combined historical data and a pre-election poll to forecast the election outcome. The forecast was based on efficient use of current poll data and historical relationships. Holbrook and DeSart [7] employed simple linear regression to predict the presidential election outcomes in the US at the state level by considering state-wide preference polls and a lagged vote variable. Jérôme et al. [9] analyzed the poor performance of the polls to forecast the defeat of the Right in the French Legislative election in 1997 and proposed a political economy model to forecast the election outcome more accurately. Lewis-Beck and Tien [12] developed a micro model for election prediction based on a survey of the voters. They showed that voters can forecast the election outcome and explained some sources of this ability along with its precision. Rallings and Thrasher [13] developed a model to estimate national party support based on voting in local elections in Britain. Stambough and Thorson [15] developed a multiple indicator model that includes the economy, challenger strength, incumbent strength and state political strength to forecast the presidential election in the US.

In this paper, Morgan Poll two-party-preferred data (www.roymorgan.com) have been chosen as a measurement indicator for the popularity of the government and the opposition in Australia. The dynamic of these data will be modelled by a mean-reverting Stochastic Differential Equation (SDE) with a term structured volatility. In addition to its constitutional right to call an election, a possibility that the government can use control tools termed as `boosts' to raise its popularity in the polls is also considered. These control tools include economic policy announcement such as tax cuts or budgets. In this paper it is assumed that only the government can use `boosts' and the opposition can do nothing.

The results of the model include the expected remaining life in power, an optimal control for the government by locating an exercise boundary which indicates whether or not a snap election should be called and whether or not a boost should be applied.

The organization of the remainder of this paper is the following. Problem formulation and notation are introduced in section 2. In section 3, volatility estimates for the poll data using EWMA (Exponentially Weighted Moving Averages) is performed and then a term structured volatility model is introduced. Numerical results in terms of the expected remaining life, call and boost exercise boundaries are given in section 4. Conclusions are presented in section 5.

## 2. Problem Formulation and Notation

The problem formulation and notation used in this paper is similar to the one in [11]. Let there be $m$ levels of popularity $S_i$ within the interval (-1,1) and $n$ time steps dividing the maximum period of $Y$ years between elections. There is also a constant lead-time $T_L$, the period between announcing and holding the election. According to the Australian constitution this lead-time must lie between 33 and 68 days and is further restricted as elections must be held on a Saturday. In the model, we set $T_L = k\delta t$ for some integer $k$. Later in the computation we set $m = 50$, $T_L = 0.12$ year (around six weeks), $n = (Y/\delta t)$ and $\delta t = 0.04$ year (around two weeks). Also, we set $-0.5 < S < 0.5$ since in reality it is very unlikely to have the value of $S$ less than -0.5 or greater than 0.5. The state variables in the model are: t, time into current term, S, the difference in the two-party-preferred popularity and B, the boost state. The maximum possible number of time steps till next election is denoted by $\psi$ (taking into account whether an election has been called or not). If the election has not been called, $\psi$ is the time until the government's term is up. If the election has been called, $\psi$ is the time until the known election date. The notations used in the model are; $V(t,S_i,B,\psi)$ : the expected remaining life at time $t$ under the government's optimal strategy, when the level of popularity is $S_i$, the total boosts remaining are $B$ $(B=0,1,\ldots,B_{max})$ and there are still at most $\psi$ periods until the election; $P_{ik}$ : transition probability from poll state $S_i$ to state $S_k$ over period $\delta t$ with no boost; $P^b_{ik}$: transition probability from poll state $S_i$ to state $S_k$ over period $\delta t$ when the government has applied a boost over period $\delta t$;

$P(W|S_j)$ : conditional probability of winning the election from true state $S_j$ ; $Q_{ij}$ : conditional probability that the true state of voting intentions is $S_i$ given that the poll state is $S_j$; $B_{max}$: the maximum number of boosts available at the beginning of the government's term in office .

Unless explicitly stated, we take $S_i$ to be the level of popularity given by the poll state at time $t$. In developing the model, the political time frame is divided into three regimes. The final time is the first regime when $\psi = 0$ (election date), the second regime is the so-called election mode, when $0 < \psi \leq T_L$ and the third regime is the non-election mode when $\psi > T_L$. In the election mode, as election date is already known, the government's only decision is either to boost or not to boost. Since the objective of the government is to maximize its time in power, the decision should maximize the expected remaining life by using boosts or not using boosts. In the non-election mode, there are more options. The government can boost and call the election simultaneously, boost but not call an election, not boost but call the election or not boost and not call the election. When there are no remaining boosts, the government can only choose whether or not to call an election. The formulation for $V(t,S_i,B,\psi)$ is derived by considering three regimes above. It is assumed that at the final time when election is held, $\psi = 0$, the newly elected government has all boosting resources renewed from $B$ to $B_{max}$. Therefore, the formulation is given by:

$$V(t,S_i,B,0) = \sum_{j=1}^{m} P(W|S_j)Q_{ij}V(0,S_j,B_{max},n) \qquad (1)$$

In the election mode, $0 < \psi \leq T_L$ , where $t$ refers to discrete time

$$V(t,S_i,B,\psi) = max\left\{\sum_{j=1}^{m} V(t+1,S_j,B-1,\psi-1)P_{ij}^{b}, \sum_{j=1}^{m} V(t+1,S_j,B,\psi-1)P_{ij}\right\} + \delta t \qquad (2)$$

In the election mode there are only two options, boost or not boost. The first summation in (2) is the option to boost, and the second one is the option not to boost. In both cases, the government remains in power up to the next time step $\delta t$ with certainty. When choosing to use boosts the transition probability is $P^b_{ij}$ which lifts the popularity to a higher level, but the number of boost remaining decreases by one. In the non-election mode, when $\psi > T_L$, the expected remaining life is given by:

$$V(t,S_i,B,\psi) = max\left\{ \sum_{j=1}^{m} V(t+1,S_j,B-1,T_L)P_{ij}^{b}, \sum_{j=1}^{m} V(t+1,S_j,B-1,\psi-1)P_{ij}^{b}, \right.$$
$$\left. \sum_{j=1}^{m} V(t+1,S_j,B,T_L)P_{ij}, \sum_{j=1}^{m} V(t+1,S_j,B,\psi-1)P_{ij} \right\} + \delta t \qquad (3)$$

The above equation contains four summations which correspond to four options available to the government in the non-election mode. The first summation is when the government is applying boosts and calling an election simultaneously while the second one is to boost but no election called. The third one is not to boost but to call for an election and the last one is when opting neither to use boost nor to call an election. In all four cases, as in the election mode, the government stays in power up to the next time step $\delta t$ with certainty. In case the government calls for an election, the number of periods until the election, $\psi$, will revert back to $T_L$ and enter the election mode.

An iterative scheme is used to determine the expected remaining life by starting with an initial estimate at time $t = 0$ and then calculate the value at final time $V(t,S_t,B,0)$ using (1) and move backward. This value becomes a boundary condition in calculating the expected remaining time in election mode in (2). Values at election mode become the boundary condition for calculating values in non-election mode in (3). Finally, the new calculated value at $t = 0$ replace the initial estimate and the procedure is repeated until it converges (the difference is less than some tolerance value).

## 3. Term Structured Volatility Model

Figure 1 (above) represent the difference in two-party-preferred between the government and the opposition in Australia. The data were taken fortnightly, however once an election is announced polls are conducted approximately weekly and even more frequently in the days leading up to the election date. The figure has similarity with the dynamic of some stock prices in finance. The two-party-preferred term refers to the distribution of preferences (votes) between the two major parties in Australia, namely the Coalition (Liberal and National Party; the government) and the Labor Party (the Opposition). In [11] the poll process was described using the following SDE:

$$dS(t) = -\mu \frac{S(t)}{1 - S(t)^2} dt + \sigma dW(t) \tag{4}$$

where $W(t)$ is a Wiener process; $S(t)$ is the difference in the two-party-preferred data between the government and the opposition $(-1 < S < 1)$ and $\mu$ and $\sigma$ are positive constants. The drift of the above SDE has a mean-reverting coefficient, always reverts to zero as in fact less popular party will react in such a way to make its popularity higher in the next poll. The constant volatility is analogous to the founding studies in the ``Black-Scholes" in option pricing in finance ([2]).

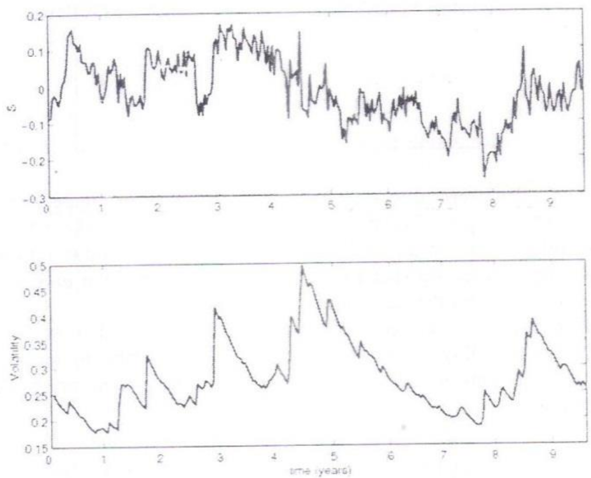

**Figure 1**. Volatility Estimates for Two-Party-Preferred Data

In order to accommodate the condition that the government can apply a boost to raise its popularity in the polls, a so-called *boost factor b* is introduced. By applying a boost, the government's popularity will rise over the next step $\delta t$ and its impact is assumed to be nonlinear. The impact of a boost will be higher when the government's popularity is at the low level than when it is already high. In [11] the dynamic of the poll process was described by:

$$dS(t) = -\mu\left(\frac{S(t)}{1 - S(t)^2}\right)dt + \sigma dW(t) + \beta dJ \qquad (5)$$

where $dJ$ is a jump process, with value *1* only upon a boost applied by the government and *0* otherwise. Using boost will give an additional rise of $\beta = b(0.5\text{-}S)$ in the government's popularity.

However, in actual fact, the poll data posses a weak time dependence, showing clustering in a similar way to the stochastic volatility models of stock price data. Therefore, a term structured volatility model is required. Volatility estimates for the two-party-preferred data from April 1993 - December 2002 are performed using the EWMA method, which is basically an example of exponential smoothing method in analyzing time series data. This method gives more weight to the recent observations and less weight to older observations in order to detect small changes in the volatility. Also, the EWMA can react to the jump in the data faster than the simple moving average method. This method has been used in the *RiskMetrics* program introduced by the American Bank JP Morgan in October 1994 to obtain estimates of volatility and correlation in the framework of Value-at-Risk (VaR) (see [6] or Chapter 57 of [16]). In relation to (4), a dynamic volatility estimate is constructed as follows.

$$\hat{\sigma}_{i+1}^2 = \lambda\hat{\sigma}_i^2 + (1-\lambda)\frac{dX_i^2}{dt_i} \qquad i = 1,2,\ldots, N \tag{6}$$

with λ = 0.94 as a weighting factor and $dX_i = dS_i + \mu\left(\frac{S_i}{1-S_i^2}\right)dt_i$ (*RiskMetrics* also used λ = 0.94). The result of this EWMA method is given in Figure 1 (below). Note that the EWMA estimate of volatility can capture the jump on the data between the fourth and fifth year. From this figure we realize that the volatility estimates for the two-party-preferred data are changing over time. This condition may be due to the rises of the volatility near the election date.

Therefore, a term structured volatility model is developed to accommodate the dynamic of the volatility in the data. This model is similar to term structure of volatility in commodity markets (see [5]). We propose a term structured volatility model as follows.

$$dS(t) = -\mu\left(\frac{S(t)}{1-S(t)^2}\right)dt + \sigma dW(t) + \beta dJ, \quad \sigma(t) = \sigma_0 + \sigma_1 e^{q(T-t)}. \tag{6}$$

**Table 1**. Parameter Estimates of Term Structured Volatility Model

| Parameters | 1993-1996 | 1996-1998 | 1998-2001 | 2001-2004 | Best Fit |
|---|---|---|---|---|---|
| $\sigma_0$ | 0.2568 | 1.2851 | 0.2096 | 0.2420 | 0.2508 |
| $\sigma_1$ | -0.0191 | -0.8720 | 0.2105 | -0.0024 | 0.1713 |
| q | 0.3968 | 0.0734 | -5.5610 | 0.9524 | -17.4800 |

In Figure 2 below, a dynamic volatility estimate is performed for each period between elections using EWMA for the last four Federal Elections. Then, a term structured volatility model is introduced in each period to capture the dynamic of the EWMA estimates. Parameters in the term structured volatility model are estimated using least-square method, while $\mu$ is estimated using the Maximum Likelihood Estimation (MLE) method. The parameter estimates of the term structured volatility model are summarized in Table 1; results for the EWMA and a term structured volatility model for each period between elections are given in Figure 2. The results seem not promising except for period 1998-2001 where the proposed model matches quite perfectly with the EWMA volatility estimates giving coefficient of determination $R^2$ of 96.96%. The condition where the volatility is rising close to the election day is also justified in this period. However, for other periods, we can say that other factors such as volatility clustering in certain time interval and jumps in the two-party-preferred data contribute to this condition. In Figure 3 the best fit of the term structured volatility estimates from the model in each period is given with the coefficient of determination $R^2$ of 51%.

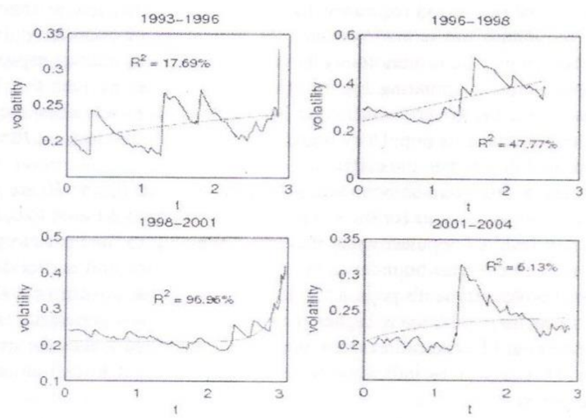


**Figure 2**. EWMA and Term Structured Volatility Model

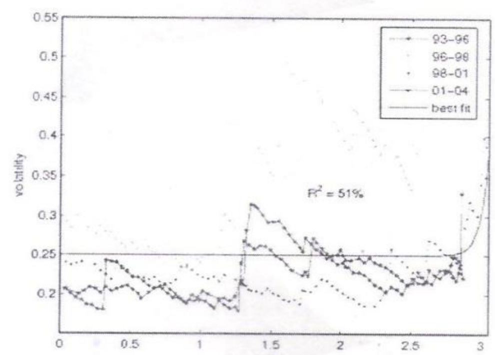


**Figure 3**. EWMA and Term Structured Volatility Model (Best Fit)

## 4. Numerical Results

Numerical results in this section are given in terms of the expected remaining life, call and boost exercise boundaries. From the data, the MLE method gives $\mu = 3.98$ and a boost factor b=0.05 is used.

In Figure 4(a) and (b), the expected remaining life for $B_{max} = 5$ and $B_{max} = 10$ are given. From both figures, in general, the expected remaining life is quite constant at the beginning of the period regardless the level of popularity and as time elapses, the expected remaining life is also constant for high level of popularity. However, the expected remaining life is monotonically decreasing as time elapses especially for low level of popularity. Comparing Figure 4(a) and (b), it can be seen that having more `boosts' will give the government longer expected life in power since the government can use them to make its popularity higher before calling an election; therefore giving a higher probability to win the election.

In Figure 5 call exercise boundaries for $B_{max} = 5$ and $B_{max} = 10$ are given. These boundaries give indications for the government about the right time to call an election given certain level of popularity and the time remaining to the next election. In these two figures, call exercise boundaries are monotone in time and earlier election needs higher level of government's popularity. But in general, the government should call an election when its popularity is higher than the opposition's popularity. Having more boosts remaining lifts the call exercise boundary higher and makes the exercise region narrower. This fact gives indication to the government not to call an early election, especially when

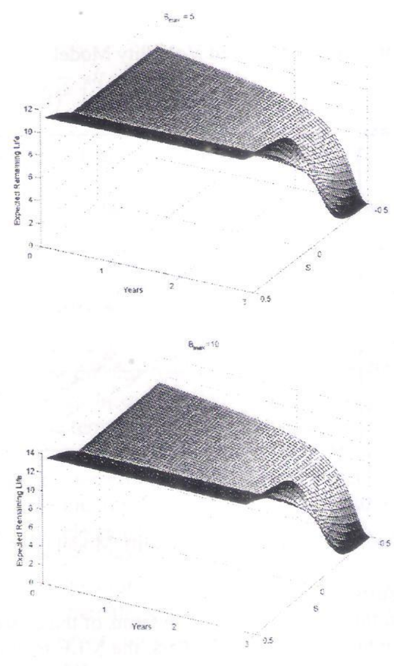


**Figure 4**. Expected Remaining Life with (a) $B_{max} = 5$ (b) $B_{max} = 10$

there are still boosts available. It is better for the government to use its boosts first before calling an election.

The option for the government to use its boosts can occur in both the election and non-election mode. Therefore, unlike the call exercise boundaries, results for boosts exercise boundaries cover the whole period of three year. Boost exercise boundaries for $B_{max} = 5$ and $B_{max} = 10$ are given in Figure 6(a) and (b). In both figures, when $B = 1$, $2$ and $3$, boost exercise boundaries are just vertical lines in the election mode. Note that in the computation, there are 3 time steps ($k=3$) in the election mode as $T_L = 0.12$ and $\delta t = 0.04$. These mean that if the number of remaining boosts is no more than k, they should be spent at the election mode regardless of the level of popularity. Applying boosts during the election mode will give optimal impacts as it close to the election day. In general, for B greater than 4, boost exercise boundaries are monotone in time and the government should apply boosts when the difference in popularity is at least greater than zero. In Figure 6(b), boost exercise boundaries are crossed over for $B = 5$, $7$ and $10$ close to the end of the period when $S$ is around zero. This means that when close to the end of the period before an election is called and $S$ is around zero and the government still has boosts to spend, they should be spent at this time and again at every time step in the non-election mode.

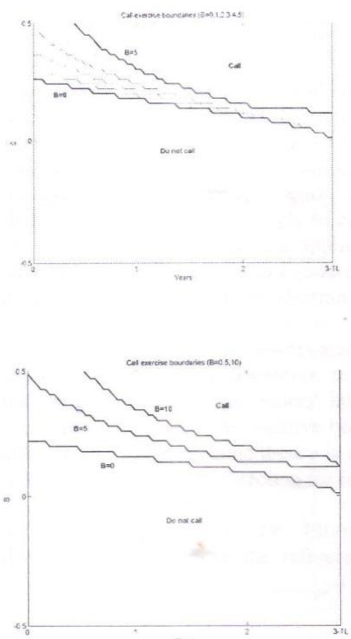


**Figure 5.** Call exercise boundaries with (a) $B_{max} = 5$ (b) $B_{max} = 10$

It is also interesting to look at call and boost exercise boundaries simultaneously as per Figure 7 for $B_{max} = 10$. It can be seen that when there are *5* or *10* boosts remaining, call and boost exercise boundaries are the same until around $t=2$ years whereupon they diverge. This means up to $t=2$ years, applying boosts and calling an election should be exercised at the same time, whereas they can be exercised separately later in the period.

## 5. Conclusions

We have given the expected remaining life in government and exercise boundaries for the model by assuming that the government can apply boosts at any time step at any level of popularity given a certain amount of boosting resource available at the beginning of the period. The expected remaining life is longer when there are more boosts available, especially during the early life of the government. However, at the final time the expected remaining life remains the same regardless of the number of boosts left at that time.

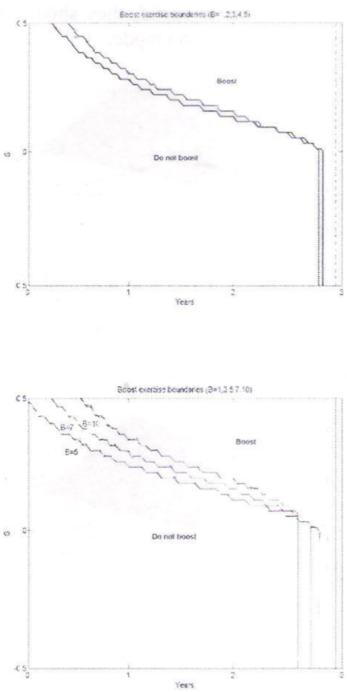


**Figure 6**. Boost exercise boundaries with (a) $B_{max} = 5$ (b) $B_{max} = 10$

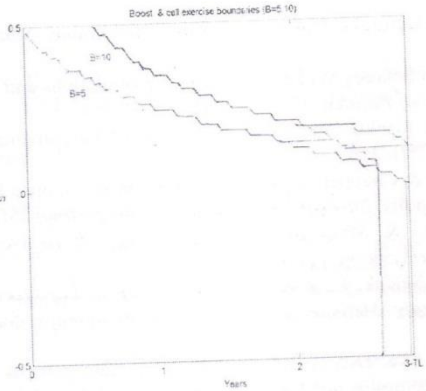


**Figure 7**. Call and boost exercise boundaries with $B_{max} = 10$

For exercise boundaries, we differentiate between call and boost exercise boundaries that both can only occur in the non-election mode. Boost exercise boundaries themselves can also occur in the election mode. These exercise boundaries are monotone in time in the non-election mode and as the number of boosts increase, the exercise boundary is lifted, giving a smaller exercise region. In terms of call exercise boundaries, this means that the government is less likely to call an early election if it still has enough boosts to spend. The same condition applies to the boost exercise boundary. When there are still boosts available, as in Figure 6(a), for $B = 2, 3$ and $4$, these should be spent at every time step during the election mode regardless of the level of popularity.

In this paper, it was assumed that only the government can apply boosts by introducing some policies or economic actions. However, in practice the opposition maintains a set of its own policy that can sway voters' intentions away from the government. These policies can be considered as negative boosts that can pull down the government's popularity. In this situation game theory is an appropriate approach to solve this problem and this will be a possible direction for further research.

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