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Volume III

Introduction

This volume of the proceedings contains papers which are classified under Statistics, Operation Research and Miscellaneous. The editors would like to express their deepest gratitude to all presenters, contributors/authors and participants of this conference for their overwhelming supports that turn this conference into a big success. While every single effort has been made to ensure consistency of format and layout of the proceedings, the editors assume no responsibility for spelling, grammatical and factual errors. Besides, all opinions expressed in these papers are those of the authors and not of the conference Organising Committee nor the editors.

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A STOCHASTIC DYNAMIC PROGRAMMING PROBLEM IN ELECTRICITY MARKETS

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Abstract. *We study a financial and operational problem relating to demand-side management in the Alberta electricity market in Canada. The problem relates to the valuation and derivation of an optimal management strategy to undertake power load curtailment subject to contractual constraints. Our solution applies stochastic dynamic programming based on the system state which includes the electricity spot price and remaining resources. The solution generates the best allocation of curtailment within a calendar year and establishes the optimal exercise boundaries, that is, when to commence and conclude curtailment.*

2000 Mathematics Subject Classification: 90C39, 91B70

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1 Introduction

In recent decades, considerable progress has been made in many jurisdictions towards deregulation of electricity markets. As a consequence a large range of innovative financial products have arisen for managing financial risk and optimizing the returns from physical assets. This paper explores a particular contract structure which provides value for a participant holding an electrical load capable of temporary curtailment.

Physical electricity markets are burdened with a requirement to maintain security and stability of the electrical network, ensure reliability for consumers and to ensure access to the market by producers. The consequent physical markets are invariably structured with relatively stringent rules and overseen by a system operator. The resulting spot (or pool) market is typically a real-time or short-term market which ensures that supply is scheduled to meet demand at any time. The inability to economically store large volumes of electrical power means that spot prices for electricity are extremely volatile, exhibiting temporary price excursions almost inconceivable in other commodity markets.

Examples of deregulated electricity spot markets are the PJM market in Northern America and the National Electricity Market (NEM) in Australia. Our investigations in this paper concentrate on the Alberta electricity market in Canada operated by Alberta Electric System Operator (AESO).

The Alberta physical electricity market is characterized as a gross pool market, meaning that all generators are compelled to sell product through the central market and each submit an offer stack representing the supply curve. The price for the entire region is set each hour by establishing the marginal cost of electricity to meet the prevailing demand according to an aggregate supply curve submitted by all available generating units. Figure 1 illustrates the hourly price for electricity over the month of January 2008. The plot clearly exhibits high volatility and price spikes. Typical prices reside at around \$50/MWh, but spikes to \$1,000/MWh are observed.

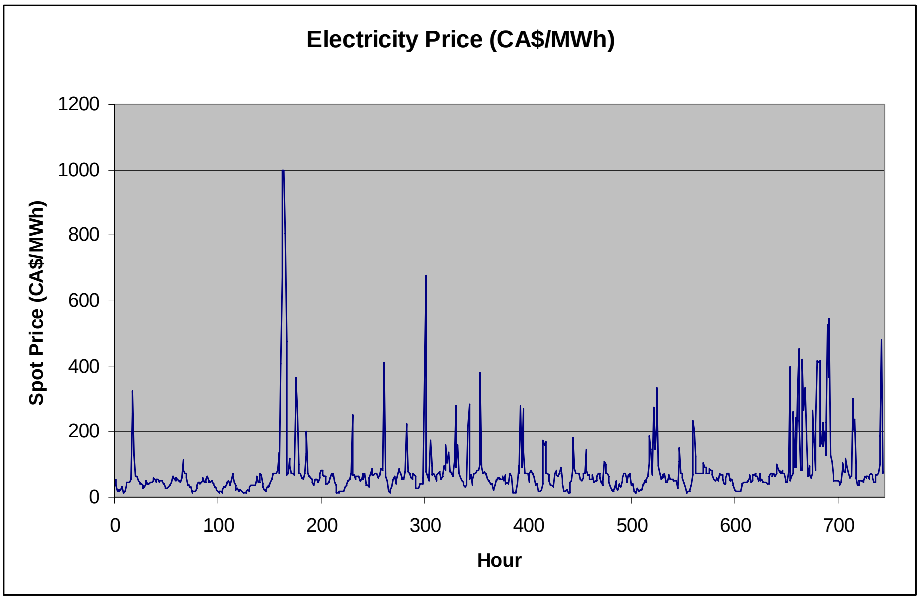

Figure 1: Electricity spot price Alberta market January 2008

To manage the financial risks arising from market price volatility, participants involved in the physical market (generators, retailers and large industrial customers) engage in hedging with derivative instruments in a secondary market (the financial market). The deregulation of electricity markets has led to innovative risk management solutions in this wholesale market. The most common hedging mechanisms involve commoditized derivative contracts written directly on the time-weighted-average spot price, such as swap contracts (also known as cash-settled forward contracts for difference) or futures contracts.

Structured financial products have been developed by participants to customize financial contracts to match the capabilities of their physical exposures.

Consider a factory which consumes electricity in a base load fashion and suppose that electricity is a dominant input cost. Suppose that the factory pays the prevailing hourly spot price for all electricity that it consumes. From time to time, the wholesale price will spike to high levels. If the factory has the capability of shutting down for those short periods, then it is capable of making considerable cost savings. It is usually the case however that physical limitations are imposed on the capability of the factory to shut down and restart. For example, there may be a lead-time to shut down processes or to restart, or there may be a minimum or maximum time for the factory to be offline.

More generally, the factory purchases electricity through an electricity retailing corporation through a contractual arrangement. Typically the electricity retailer will purchase wholesale electricity through the spot market and will on sell to the customer for a fixed price tariff. If the retailer is able to convince the customer to undertake curtailment during the periods of high wholesale prices, then the retailer can enhance margins.

Economically, high electricity prices are a transparent signal to consumers representing a scarcity of resources for production or transmission. Consumers who are willing to reduce consumption at these valuable times should be rewarded.

Our analysis is then to establish what is the *fair value* for the discount that a retailer should offer to a customer in return for the rights to curtail the load. The rights to call curtail events will be subject to constraints (for example, a limited number of calls, or a limited number of hours in a curtailed state, or a delay between calling for the curtailment and the shutdown of the factory). The particular contract specifications are detailed below.

Consider a financial contract between two parties A and B. Party A buys a firm volume of *V* megawatts (MW) of electricity at a fixed cost in each hour across a calendar year at a price *P* dollars per megawatt hour ($/MWh) from Party B.

Party B possesses a right to call curtailment events from Party A whereupon Party A must source its electricity directly from the spot market at the prevailing market price (of course it maintains the alternative to shut down its operations).

The mechanics of a curtailment event and associated constraints are listed below.

- Party B will notify Party A of its desire to initiate a curtailment event.
- After a statutory delay of *D* hours, the curtailment will commence (that is, the fixed price contract is temporarily suspended).
- Party B will notify Party A of its desire to conclude the curtailment event.
- After a statutory delay of *E* hours, the curtailment will conclude (that is, the fixed price contract will be reinstated).
- The aggregate time in a curtailed state over the calendar year must be no more than *H* hours.

Typically the periods *D* and *E* depend on the physical processes of the customer.

Of course, it is sensible for party B to call curtailment events only when the spot price is high. The optimal strategy for Party B is to recognize how high the spot price should be to warrant calling a curtailment event, taking into account the number of hours of remaining curtailment. The amount of additional savings achieved by Party B should be reflective in a reduction in price which is offered to Party A.

For our particular case study, we will consider a contract of a notional 1 MW volume, with 100 hours of curtailment in each year and with a two hour delay to commence curtailment but no delay to reinstate the contract.

The remainder of this paper is organized as follows. In section 2, we discuss the literature review related to this paper containing swing options and its method of valuation. Our model is given in section 3 along with its assumptions and stochastic dynamic programming algorithm. In section 4, we give results of our model in terms of value of the option and curtailment boundaries. We conclude our paper in the last section.

2 Literature Review

The contract structure detailed in section 1 is part of a broader class of options termed *swing options*. This class of derivative allows the holder the right to call the option on a number of specified exercise dates, allowing the buyer the ability to 'swing' the price of the underlying asset as described in Geman (2005).

The literature contains a range of such contract structures and various approaches to valuation and management of the contract. The contracts are particularly prevalent in gas and electricity markets where storability of commodities is limited and there is continuous production and consumption of the product.

Methods used in the valuation include binomial and trinomial trees, simulation, PDEs and Stochastic Dynamic Programming (SDP) (see for example Lari-Lavassini, Simchi and Ware (2001), Keppo (2004), Ware (2007), Baldick, Kolos and Tompaidis (2006)).

The field of option pricing has a vast literature of theoretical and practical works and considerable efforts in financial mathematics have been devoted to developing a rigourous foundation. It is now well accepted that the fair value of derivatives is established through calculating conditionally expected payoffs under risk-neutral measures (Wilmott (1998)).

However, the bulk of the literature is developed under the *complete* market framework where derivatives can be hedged with other securities. In the situation of electricity derivatives such as the contract under consideration, the non-storability of electricity as a commodity means that there is no perfect hedging strategy available. The common consensus then, is that the fair value of contingent claims are then established by calculating their payoff under real-world expectations and then possibly applying a premium or discount for the risk transfer at a market price of risk (Wilmott (1998) and Geman (2005)).

In the current problem, we establish the fair value of the contract by deriving the expected payoff under the price dynamics of electricity spot prices using the real-world measure.

3 The Model

3.1. Modelling Assumptions

We make the following assumptions on the electricity price dynamics and how they are modeled, and the consequences for managing the contract.

- Electricity spot prices obey a Markov process. The dynamics of the spot price have been calibrated to historical spot price outcomes and prices obey a homogeneous transition process.
- The control strategy is undertaken based on the *state* of the system, where the state is defined by:
    - The number of hours into the year
    - The prevailing electricity spot price,
    - Whether the contract is currently in a state of curtailment or not curtailed,
    - The amount of curtailed hours used already and the amount of curtailable hours available for rest of year.
- There is no forward-looking capability apart from the current spot price, and a known statistical behaviour from that spot price. For instance, if today is a hot day, the method does not know specifically that tomorrow is a hot day, but that it is statistically likely given that spot prices are high today.
- We assume an hourly resolution for the modelling, that is, the decision process is performed at each hour.
- We neglect the time value of money in the analysis, but it becomes an important consideration in applications in industry, particularly during periods of high interest rates.

It is evident that the electricity pool prices exhibit a diurnal pattern and annual seasonality. More sophisticated models of the price dynamics are used in industry to account for this behaviour, which may be important for some contract structures. The behaviour can be modeled by either modifying the transition probabilities or in the discretisation of price states.

3.2. Primary Methodology

The fair value of the curtailment optionality is the expected payoff for the contract under an optimal strategy of curtailment less the expected payoff for the contract under the alternative of never curtailing. The optimal strategy is defined as the set of state-dependent decisions to commence or conclude curtailment, which maximizes the expected payoff of the contract.

The primary modeling methodology is based on a *stochastic dynamic program* (also termed *optimal stochastic control* or *Markov decision process*). Essentially this method is a tree based approach which establishes from each state of the system whether the pool price is sufficiently high to warrant invoking a curtailment event given the amount of resource (number of curtailable hours) remaining in the inventory. Analogously, if the contract is in a curtailed state, the method establishes whether the spot price has fallen sufficiently low to warrant reinstating the contract. Swing options are predominantly valued using this methodology.

The solution of the dynamic program yields both the optimal exercise *strategy* and the *value* of the curtailment. The optimal exercise strategy can be implemented on simulated price paths to derive a range of possible curtailment value, given the variability of pool price outcomes.

3.3. Stochastic Dynamic Program Algorithm

This section makes explicit the solution method which has been implemented to value the curtailment contract and to establish the optimal exercise behaviour.

Let

- dt = time step in hours (1 hour for the hourly Alberta market)
- K = fixed price in $/MWh for contract during periods when not curtailed
- C = contract volume in MW
- Y = total length of the curtailment period (one year = 8760 time steps)
- t = time steps into the current year
- H = total inventory of curtailment hours under contract (100 hours)
- h = total number of curtailment units used to date
- M = indicator variable
  - $= 0$ if system is in an uncurtailed state (normal operating)
  - $= \infty$ if system is currently in a curtailed state
  - $= m$ if system is uncurtailed, but curtailment has been called and will start in m timesteps
  - $= -m$ if system is curtailed, but conclusion to curtailment has been called and will end in m timesteps
- S = spot price state (discretised set of possible spot prices)
- J = number of spot prices in discretisation $(S_1, \ldots, S_J)$
- D = delay of time steps from calling curtailment until it commences (two hours).
- E = delay of time steps from calling end to curtailment until it concludes (zero hours)
- $P(S_1,S_2)$ = probability that the spot price will make a transition from price state S_1 to price S_2 over time step dt at time t.

Define the state of the system by the tuple: $X = (t, S, h, M)$ and let the value remaining in the curtailment contract be $V(t, S, h, M)$. We present the value function at state X dependent on the value functions at the possible next state X'. Over that time step, we assume that the optimal curtailment control is applied to maximize the total value. The possible controls available are to do nothing, to curtail from an uncurtailed state, or to uncurtail from a curtailed state.

The terminal condition states that when the end of the year is reached ($t = T$), there is no more value to be extracted from curtailment because time has 'run out':

$$V(T, S, h, M) = 0 \qquad \text{for all } S, h, M$$

The boundary condition on remaining resources H dictates that when the total hours available for curtailment have been used ($H = 0$), then no more value can be extracted from the contract:

$$V(t, S, 0, M) = 0 \qquad \text{for all } t, S, M$$

We have the following 4 regimes:

- (i) system is not curtailed ($m = 0$),
- (ii) curtailment has been called but has not yet started ($m > 0$),
- (iii) curtailment is in place or ($m = \infty$)
- (iv) curtailment is in place but conclusion has been called ($m < 0$)

The dynamic program under each regime is presented below. The recursion relation states that the value in the next time step is the value which is extracted from any curtailment in the current time step, plus the expected value of the contract after the next time step, conditioned on the uncertain movements in spot prices.

$$V(t, S, h, 0) = \max\left\{ \sum_{j=1}^{J} V(t+dt, S_j, h, 0) P(S, S_j), \quad \sum_{j=1}^{J} V(t+dt, S_j, h, D) P(S, S_j) \right\}$$

$$V(t,S,h,m)=\sum_{j=1}^{J}V(t+dt,S_j,h,m-1)P(S,S_j) \quad \text{for } m>1$$

$$V(t,S,h,1)=\sum_{j=1}^{J}V(t+dt,S_j,h,\infty)P(S,S_j)$$

$$V(t,S,h,\infty)=\max\left\{\begin{array}{l}\left(\sum_{j=1}^{J}V(t+dt,S_j,h+1,\infty)+(S_j-K)Cdt\right)P(S,S_j),\\ \left(\sum_{j=1}^{J}V(t+dt,S_j,h+1,-E)+(S_j-K)Cdt\right)P(S,S_j)\end{array}\right\}$$

$$V(t,S,h,-m)=\left(\sum_{j=1}^{J}V(t+dt,S_j,h+1,-m+1)+(S_j-K)Cdt\right)P(S,S_j) \quad \text{for } m>0$$

4 Results and Discussion

The SDP is applied on a grid of J = 31 spot prices uniformly distributed in a log scale between \$8/MWh and \$1,000/MWh. We have calibrated that the fixed price is K = \$61/MWh. The calculation is performed on a notional volume C = 1 MW.

Figure 2 shows the value of the curtailment contract related to our model. It can be seen that the value is monotonically decreasing in time. That means in the beginning of the time, the value of the curtailment contract is more expensive than when the time close to maturity. This finding is in line with option value in finance, that is, the option is more expensive when it is still far away from maturity.

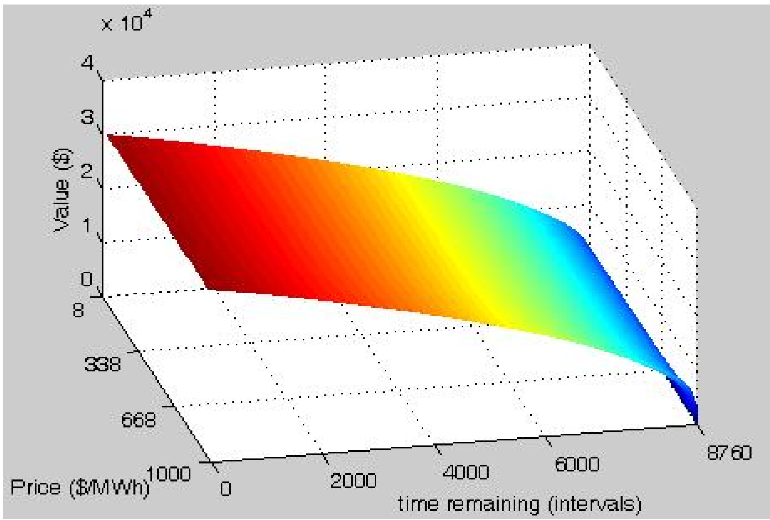

Figure 2. Value of the Curtailment Contract

In figure 3 we give curtailment call boundary when there are still 40 hours and 80 hours remaining for curtailment. From both boundaries, in general, we can see that when price is high enough (upper region value, red) it is better to call for curtailment, so the decision should be made to call the curtailment event into action. The curtailment call boundaries are in general monotonically decreasing in time, meaning that earlier in the period, the decision to call the curtailment event is made when the price is higher than when in the later period. When we compare the effect of the number of hours remaining for curtailment, it is clear that having fewer hours remaining will make the boundaries higher. This is reasonable since when there still more hours remaining for curtailment, it is better to use them when the price is higher, giving the higher boundaries.

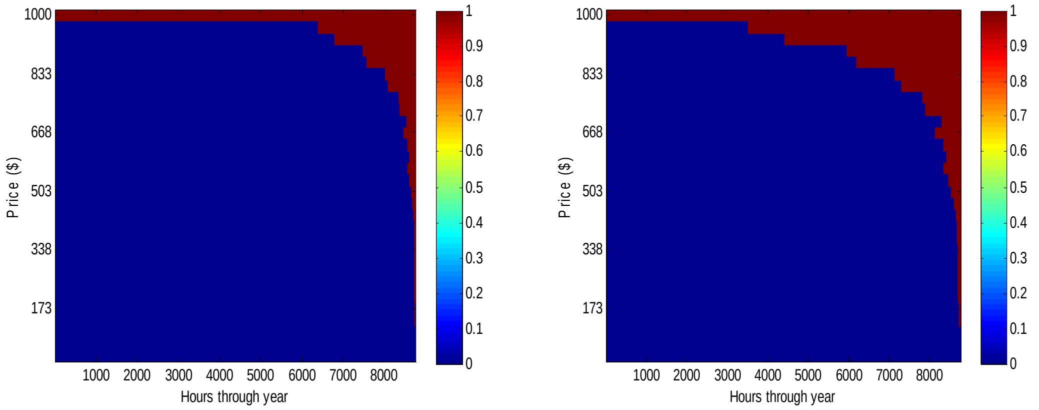

Figure 3. Curtailment call boundary when there are 40 hours (left) and 80 hours (right) of curtailment remaining

In Figure 4, curtailment call end boundaries for 40 hours and 80 hours of curtailment remaining is given. From these figures, when the price is low (red) the decision to call an end to the curtailment should be made, while when the price is high (blue), no action should be taken. Comparing between 40 hours and 80 hours of curtailment remaining in term of curtailment call end boundaries gives the same analysis as in Figure 3. When there are still 80 hours remaining for curtailment, the decision to call an end to the curtailment should be made on the price which is relatively lower than when there are still 40 hours remaining for curtailment. That means the curtailment call end boundaries is higher when there are still less hours remaining for curtailment.

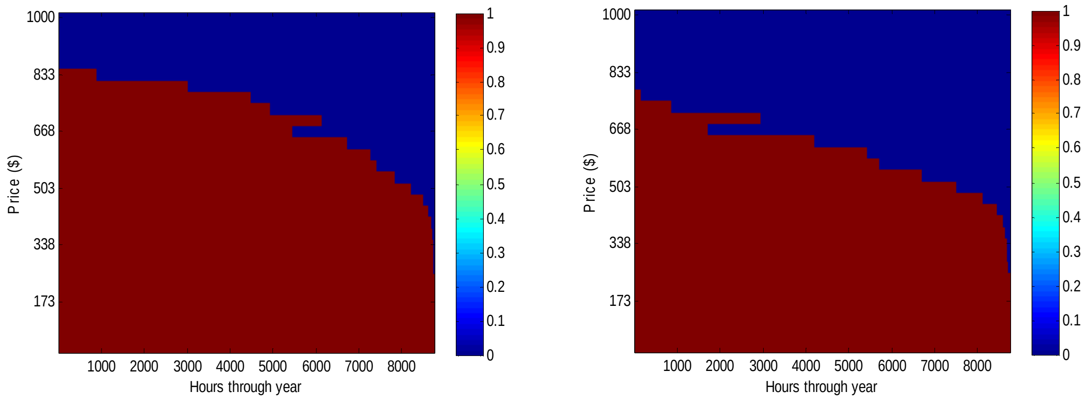

Figure 3. Curtailment call end boundary when there are 40 hours (left) and 80 hours (right) of curtailment remaining

5. Conclusions and further research

The analysis contained in this article has provided a methodology and results for the valuation of a curtailment contract which has common applications in the electricity industry. Recent releases (Letourneau (2008)) by the AESO promote demand side response and curtailment from users within the Alberta electricity grid. Financial structures like the one analysed in this article provide incentives to entice such behaviour. The methodology applied is a relatively standard approach based on the current practices for fair valuation of derivatives and stochastic dynamic programming. Electricity exhibits the characteristic of nonstorability meaning that delta hedging strategies are unavailable. As a consequence, the eventual holder of this financial product is subject to a residual financial risk. It is intended in our future work to quantify the financial risk profile of this style of curtailment contract. While this paper presents the optimal exercise strategy and the expected payoff from the instrument, industry would benefit from an understanding of distribution of payoffs for portfolio engineering. The calculation method used in the present paper is highly computational intensive. Enhancements are planned which may reduce the computation time for deriving long-term curtailment strategies on markets with very short-term price dynamics.

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