UPM
UNIVERSITI PUTRA MALAYSIA
BERILMU BERBAKTI

Proceedings of the 4th International Conference on Research and Education in Mathematics

"Meeting Challenges of Global Research and Education in Mathematical Sciences"

Renaissance Hotel Kuala Lumpur, Malaysia
21st - 23rd October 2009

Jointly Organized by :

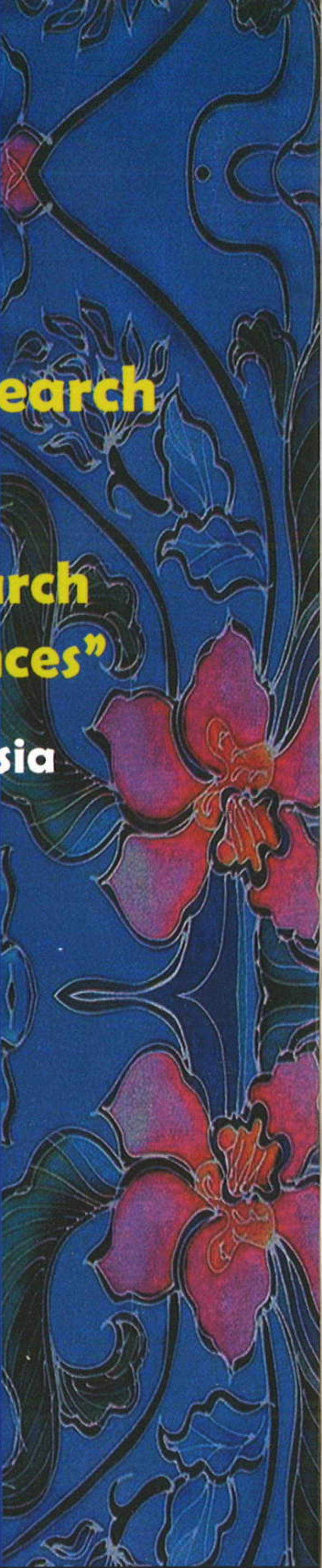
Institute for Mathematical Research,
Universiti Putra Malaysia (INSPEM)

Institute of Mathematics,
Vietnam Academy of Science & Technology (IMVAST)

Faculty of Mathematics & Natural Sciences,
Bandung Institute of Technology, Indonesia (ITB)

Malaysian Mathematical Sciences Society (PERSAMA)

Malaysian Society for Cryptology Research (MSCR)



Proceedings of the 4th International Conference on Research and Education in Mathematics

"Meeting Challenges in Global Research and Education in Mathematical Sciences"

21-23 October 2009
Renaissance Hotel Kuala Lumpur, MALAYSIA

Editors
Mohamad Rushdan Md. Said, Hishamuddin Zainuddin, Noor Akma Ibrahim,
Rohani Ahmad Tarmizi and Habshah Midi

Jointly Organized by:

Institute for Mathematical Research,
Universiti Putra Malaysia (INSPEM)

Institute of Mathematics,
Vietnam Academy of Science & Technology (IMVAST)

Faculty of Mathematics & Natural Sciences,
Bandung Institute of Technology, Indonesia (ITB)

Malaysian Mathematical Sciences Society (PERSAMA)

Malaysian Society for Cryptology Research (MSCR)

Supported by:

Abdus Salam International Centre for Theoretical Physics (ICTP)

United Nations Educational, Scientific and Cultural Organization (UNESCO)

Institute for Mathematical Research
Universiti Putra Malaysia

Published in Malaysia by:
Institute for Mathematical Research (INSPEM)
Universiti Putra Malaysia
43400 UPM Serdang
Selangor

Typeset: Nor Azlida Aminudin, Zurita Ismail and Zahratun Nur Yosminar

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

International Conference on Research and Education in Mathematics
(4th: 2009: Kuala Lumpur).
Meeting challenges in global research and education in mathematical sciences: proceedings of the 4th International Conference on Research and Education in Mathematics, 21-23 October 2009, Renaissance Hotel, Kuala Lumpur/ editors Mohamad Rushdan Md. Said...[et al.]; jointly organized by: Institute for Mathematical Research, Universiti Putra Malaysia...[et al.]; supported by Abdus Salam International Centre for Theoretical Physics (ICTP), United Nations Educational, Scientific and Cultural Organization (UNESCO).
ISBN 978-967-344-092-4
1. Mathematics--Congresses. 2. Mathematics--Research--Malaysia--Congresses I. Mohamad Rushdan Md Said 2. Universiti Putra Malaysia. Institut Penyelidikan Matematik. 3. Abdus Salam International Centre for Theoretical Physics IV. UNESCO VI. Title
510.72

PREFACE

The International Conference on Research and Education in Mathematics (ICREM) is a biennial conference organized by the Institute for Mathematical Research, Universiti Putra Malaysia. The fourth conference (ICREM4) differs from the earlier ones by having joint organizers from abroad namely Institute of Mathematics, Vietnam Academy of Science and Technology and the faculty of mathematics and natural Sciences, Bandung Institute of Technology, Indonesia as well as local ones i.e. Malaysian Society Mathematical Sciences and Malaysian Society for Cryptology Research. The conference is also supported by Abdus Salam International Centre for Theoretical Physics, Trieste, Italy and United Nations Educational, Scientific and Cultural Organization (UNESCO).

The present proceedings capture part of the excitement of the conference documenting well over one hundred papers contributed by participants from more than twenty countries. They cover all four main areas in mathematical sciences i.e. pure mathematics, Applied Mathematics and Theoretical Sciences, Statistics and Mathematics Education but ones that mirror the interests of the regional community of mathematical scientist and practitioners.

The Organizers would like to thank all the invited speakers and participants for their contributions in making this conference a success and hence brought forth this valuable proceedings.

We would like to express our deepest appreciation to all sponsors of the conference, without which the conference may not be realized. Specifically, we would like to mention Abdus Salam International Centre for Theoretical Physics UNESCO, Ministry of Higher Education and Ministry of Science, Technology and Innovations, Malaysia. Last, but not least, our utmost thanks to the management of the University for their unfailing support towards our activities.

CONTENTS

PLENARY AND INVITED SPEAKERS

MATHEMATICS EDUCATION

STATISTICS

PURE MATHEMATICS

APPLIED MATHEMATICS

COMMODITY PRICE AND VOLATILITY PRICE MODELS OF INDONESIA MARKET

[1]Ferry J. Permana, [2]J. Dharma Lesmono, [3]Erwinna Chendra
[1,2,3]Department of Mathematics, Parahyangan Catholic University, Bandung 40141, Indonesia
e-mail: [1]ferryjp@home.unpar.ac.id, [2]jdharma@home.unpar.ac.id, [3]erwinna@home.unpar.ac.id

Abstract. *Commodity markets in Indonesia have experienced a significant growth. Nowadays, commodities, e.g., palm oil and gold, are also traded in the exchange as financial instruments, i.e., futures contracts, as well as physical assets. Commodity price and volatility models are being crucial for forecasting the price and developing risk management tools. A stochastic model called the potential diffusion model is proven to be suitable to model the commodity price. In the spirit of market equilibrium, such a model is more realistic, even applied to the price with multiple attraction regions. The potential diffusion model can capture the characteristic behavior of the price. It can also be used to derive a volatility model, which is more realistic than a constant volatility assumed by most commodity models such as a geometric Brownian motion or a mean-reversion diffusion model. We applied the potential diffusion model to prices of commodities traded in Indonesia market. As a performance measure, we compare the distributional characteristics of the original price to those of simulated prices based on the potential diffusion model. For the volatility model, comparison is made between the daily volatilities based on the potential diffusion model and the historical volatilities.*

2000 Mathematics Subject Classification: 62P05, Secondary 91B70.

Key-words: *commodity price model, commodity volatility model, potential diffusion model.*

1 Introduction

Commodities suround us everywhere in our daily life. People consumes wheat or rice, vegetables and fruits as foods, uses cotton to make clothes, metals and wood for making tools and machinery, and also needs oil, gas or electricity as energy resources. In the past, commodities are traded in the spot markets. Buyers and sellers meet to make transactions for immediately delivery. Nowadays, the commodity markets have explosively grown. In the modern markets, the so-called exchanges, buyers and sellers do not necessarily meet to make transactions. Besides traded as physical assets, commodities are also traded as financial instruments, such as futures and options.

The palm oil and gold futures contracts have been traded in Jakarta Futures Exchanges (JFX) since its opening in 2000. The palm oil is particularly being the strategic commodity for Indonesia economics. Indonesia has taken an advantage from biofuel campaign by being a leader of palm oil producer since 2006. Our research is devoted as contribution to the Indonesia commodity market. Commodity price fluctuates randomly and makes the market participants bear the market risks. Modeling the commodity price is being essential for forecasting and also for developing the risk management tools. Hence, we intend to model the palm oil and the rolling gold price dynamics. It is interesting to compare the behavior of those commodity price dynamics since they will have different characteristics: palm oil is a consumption asset, on the other hand gold is an investment asset.

Figure 1. Rolling Gold log spot prices
(Jan ‘03 – Nov ‘08)

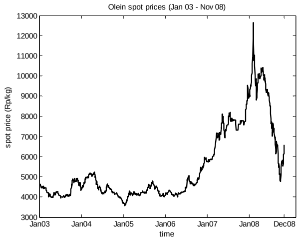


Figure 2. Palm oil log spot prices
(Jan ‘03 – Nov ‘08)

We will work on the rolling gold and the palm oil log prices of Indonesia market, i.e., the JFX. The palm oil spot price is quoted in Rupiah per kilogram and and the rolling gold spot price is quoted in Rupiah per gram. Note that, in this paper everywhere we will work on the log spot prices. The palm oil and the rolling gold log spot prices over the period of January 2003 – Nov 2008 are given in Figures 1 and 2. Those price are highly correlated with correlation coefficient of 0.78.

In our previous work (see [14] and [15]) we found the presence of linear trend in the rolling gold dynamics. We tested three stochastic models: Geometric Brownian Motion (GBM), mean-reversion diffusion and potential diffusion models to model the stochastic component of the dynamics price. Modeling commodity price is also essential for risk management, e.g., Value-at-Risk, derivatives pricing or portfolio management. The potential diffusion model assumes a constant volatility over a period of time. In practice, that is not the case. It can be shown ([1]) that we can extend the potential diffusion model to develop the dynamics of volatility price. We apply such a model to the palm oil and rolling gold spot prices. The obtained implied-model volatility will be compared to the realized volatility.

2 Commodity price models

The standard approach to model the commodity spot price $F(t)$, or more often its logarithm $Y(t) = \log F(t)$ on each day t, is to decompose it as the sum of the deterministic $s(t)$ and stochastic components $X(t)$:

$$Y(t) = s(t) + X(t), \tag{1}$$

as suggested by most researchers and practitioners working on commodity markets ([11], [12], [17, [19]). The deterministic component is commonly represented by trend, e.g., due to inflation, and/or seasonality, e.g., due to the harvest cycle for agricultural commodities or wether condition for the energy commodities. Trigonometric functions (sine and cosine) are often proposed to model seasonality. Our investigation using graphical techniques: run sequence plot, seasonal subseries plot and autocorrelation plot shows that both the rolling gold and the palm oil prices do not exhibit seasonality.

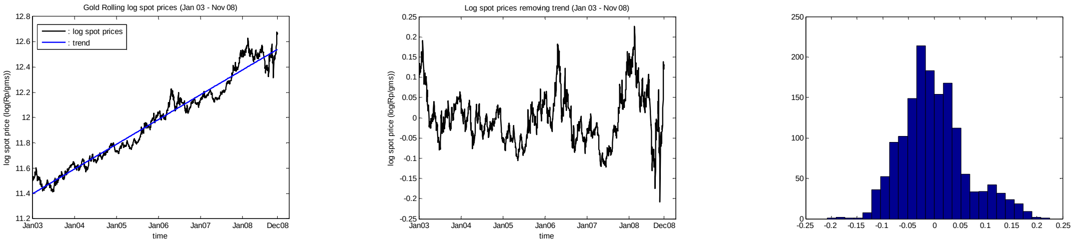


Figure 3. Log spot prices vs. linear trend (*left*), detrended log spot prices (*center*) and the histogram of detrended log spot price (*right*) of the Rolling Gold (Jan '03 – Nov '08)

At a glance we can see the presence of linear trend on the rolling gold log spot price dynamics from Figure 1, but it does not occur on the palm oil log spot prices. To investigate presence of linear trend, we apply the least square method on the rolling gold log spot prices and the obtained coefficient determination is about 96 %. In Figure 3 (left) we include the linear trend. Plot of the detrended log spot price shown by Figure 3 (center) will be considered as the stochastic component.

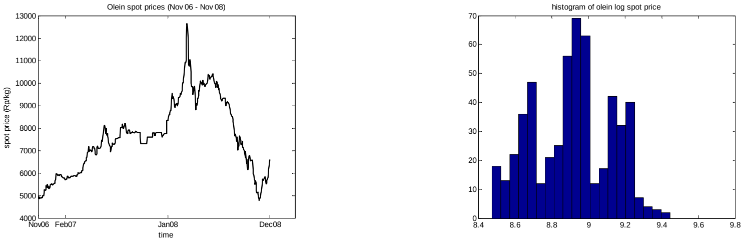


Figure 4. Log spot prices (*left*) and its histogram (*right*) of the palm oil
(Nov '06 – Nov '08)

Starting from the mid of 2006, the oil price climbed up and reached the peak in the beginning of 2007. It has affected the palm oil and the rolling gold prices as shown more clearly by the palm oil in Figure 2. For the palm oil, we will focus to model its price dynamics over period of November 2006-November 2008 whose plot is given in Figure 4 (left) and its histogram is presented in Figure 4 (right). Since the palm oil price does not exhibit trend and seasonality, we will consider it as the stochastic component.

2.1 Stochastic Models

The classic model of the stochastic component is the geometric Brownian motion (GBM). At first it is used to model the stock price. In 1976, Black ([3]) used it to model the futures commodity price dynamics. Based on the GBM model, Black, Scholes and Merton introduced the famous option price model, i.e., the Black-Scholes model. The asset price process following the GBM can be represented by the following stochastic differential equation:

$$\frac{dF(t)}{F(t)} = \mu dt + \sigma dW(t)$$

where $F(t)$: the asset price, e.g., the spot price of commodity, at time t, μ: the expected return of asset, σ: the volatility and $dW(t)$: the increment of Wiener process. By setting $G(F(t),t) = \log(F(t))$ and applying the Ito's Lemma, we obtain the process of $F(t)$ will follow the stochastic differential equation:

$$d(\log(F(t))) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW(t) \tag{1}$$

Equation (1) means that the log return, that is $\log(F(t+dt)/F(t))$, is normally distributed with mean of $(\mu - \frac{1}{2}\sigma^2)dt$ and variance of $\sigma^2 dt$.

In practice, it is often that the GBM is not suitable to describe the commodity price dynamics. The price will leave the mean level to go up or go down because of the unbalanced supply and demand, e.g., because of overproduction or a shortage. In equilibrium market setting, the price will eventually return towards the mean level after the event goes away and the supply and demand are balanced. Hence, modeling the commodity spot price using a mean-reversion model is more realistic than the GBM model.

The mean-reversion diffusion model, introduced by Vasicek to model the random evolution of interest rates, is widely incorporated in energy and agricultural commodity price modeling ([2], [5], [6], [11] and [14]). The asset price process following the mean-reversion diffusion model can be represented by the following stochastic differential equation:

$$d\left(\log(F(t))\right) = \alpha\left(m - \log(F(t))\right) dt + \sigma dW(t) \tag{2}$$

where α: the mean-reversion rate, m: the mean-reversion value and $dW(t)$: the increment of Wiener process.

2.2 Potential Diffusion Model

Our investigation showed that the log returns of the rolling gold and the palm oil are not normally distributed. It indicates that the GBM is not appropriate model for both the rolling gold and the palm oil. Simulation studies also showed that the generated log prices obtained by the GBM model cannot capture the skewness and kurtosis of the original log spot prices of both the rolling gold and the palm oil (see [15] and [16]).

In practice, it is often that the price clustering over a long observation period concentrates in a number of attraction regions. It is shown very briefly by the palm oil as given by histogram in Figure 4 (right) whose at least three attraction regions: 8.7, 8.9 and 9.2. It means that the price moves between three attraction regions. Multiple attraction regions occur since the current mean level is different from the previous mean level. The potential diffusion model can deal with such a situation. It contrasts with the mean-reversion diffusion model which can only deal with a single attraction region. We should note that that the mean-reversion diffusion model is a special case of the potential diffusion model with a constant reversal rate.

Modeling the commodity price using a potential diffusion model has introduced by Borovkova et. al. ([4]). The price process following a potential diffusion model is represented by the following stochastic differential equation:

$$dX(t) = -V'(X(t))dt + \sigma dW(t), \tag{3}$$

where $X(t) = \log(F(t))$, $V: R \to R$ is a twice continuously differentiable function such that $V(t) \to \infty$ as $|x| \to \infty$ and $\int_{-\infty}^{\infty} \exp\left(-\frac{2V(X(t))}{\sigma^2}\right) dx < \infty$. Those conditions assure that the invariant distribution of the process $(X(t))$ is a Gibbs distribution with density

$$\pi_\sigma(x) = \exp\left(\frac{2V(x)}{\sigma^2}\right) \tag{4}$$

(for proof see e.g., [13]).

The relationship (4) means that there is a one-to-one correspondence between the invariant distribution of the process and the diffusion's drift, given by the potential.

The potential $V(X(t))$ can be estimated, together with the volatility σ, from historical data by first estimating

$$G_\sigma(x) = \frac{2}{\sigma^2}U(x) = -\log(\pi_\sigma(x)) \tag{5}$$

by

$$\hat{G}_\sigma(x) = -\log(\hat{\pi}(x)),$$

where $\hat{\pi}$ is some estimate of the observations' marginal density, e.g., a kernel density estimator or a histogram smoothed by a polynomial or a sum of Gaussian densities. It is the density function of the so-called the Gibbs distribution.

Discrete-time model of the stochastic differential equation (3) obtained by applying the Euler Scheme is given by following equation:

$$X(t_i) - X(t_{i-1}) = -U'(X(t_{i-1}))\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon_{t_i},$$

where ε_{t_i}: standard normal random variable, $X(t_i) = \log(P(t_i))$ and Δt: the unit of time step.
The parameter estimates of the model can be obtained by applying the least square method.

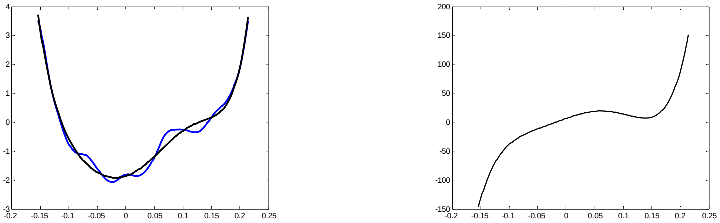


Figure 5. The 6th degree polynomial potential (*left*) and the reversion rate (*right*) of the stochastic component of the rolling gold (Jan '03-Nov '08)

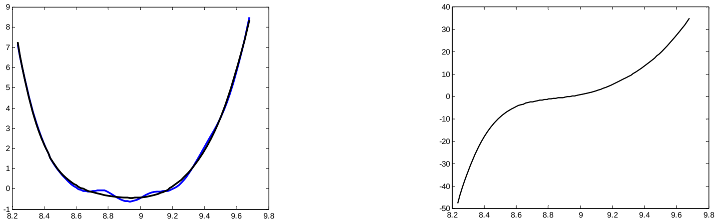


Figure 6. The 6th degree polynomial potential (*left*) and the reversion rate (*right*) of the stochastic component of the palm oil (Nov '06-Nov '08)

Using the potential diffusion model, we estimated the model parameters for stochastic component of the rolling gold and the palm oil log spot prices. The potential function is estimated by fitting a 6th degree polynomial to the histogram of log spot prices as shown by Figure 5 (left) and Figure 6 (left). The reversal rate is described by the functions given in Figures by 5 (right) and 6 (right). The estimated values of σ are 0.013 per day (21.98 % per annum) and 0.082 per day (or 45.22 % per annum) respectively to the rolling gold and the palm oil.

To investigate performance of those models we will refer to indicators proposed by Geman ([7]):

- From trajectorial standpoint – the trajectories generated, e.g., by the Monte Carlo methods, must on average look like the observed ones.
- From a statistical standpoint – the moments of the distribution of $S(T)$ (for $T > 0$) must coincide with empirical moments, at least the first four moments (i.e., mean, variance, skewness and kurtosis).

Using the obtained parameter estimates, we generate 1000 paths whose follow the models. To investigate performance of the model, we compare the first four moments (mean, standard deviation, skewness and kurtosis) of the generated paths to those of the original price. The result presented in Table 1 shows that the generated log price obtained by the potential diffusion model can fit mean, standard deviation and kurtosis of the original log price. The skewness of the generated log price is not too close to that of the original log price but it is much closer that those obtained by the GBM and the mean-reversion models. For comparison, the skewness of the generated log price are -0.0132 and 0.0441 respectively obtained by the GBM and the mean-reversion model for the rolling gold and 0.0243 and -0.4881 respectively obtained by the GBM and the mean-reversion model for the palm oil. We also present one realizalition path vs. the original price in the same coordinate plane in Figure 7 to show that the trajectories generated, i.e. the generated log price, must on average look like the observed ones, i.e. the original log price.

Table 1. The first four moments of the original and the generated log price

Moment	rolling gold (Jan ‘03 – Nov ‘08)		palm oil (Nov ‘06 – Nov ‘08)	
	original log price	generated log price	original log price	generated log price
mean	11.9684	11.9716	8.9109	8.8199
std. deviation	0.3364	0.3295	0.2138	0.1903
skewness	0.1726	0.0544	-0.041	-0.0601
kurtosis	1.8622	1.8215	2.2124	2.5527

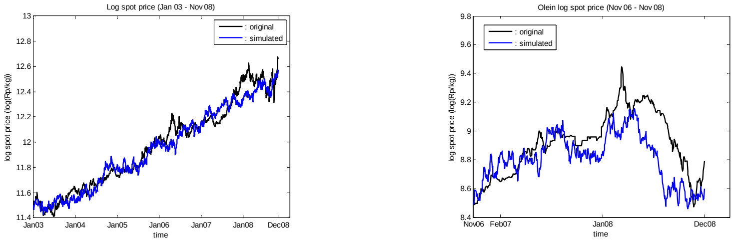


Figure 7. Original log price vs. generated log price of
the rolling gold (Jan ‘03-Nov ’08) (*left*) and the palm oil (Nov ‘06-Nov ‘08) (*right*)

3 Model-implied volatility of commodity price

Our simulation study has shown that the potential diffusion model performs better than both the geometric Brownian motion and the mean-reversion diffusion models. The generated price obtained by the potential diffusion model can fit the first four moments of the original path better than those obtained by the geometric Brownian motion and the mean-reversion diffusion models. So far, applying the potential diffusion model means to assume a constant volatility parameter σ. In practice, that is not correct. As a striking example, we look at the palm oil log spot price over period January 2003 – November 2008 whose plot is given in Figure 2. Permana et.al ([14]) has shown that the volatility of the palm oil log spot price implied the potential diffusion model over period January 2003 – October 2006 is lower than that over November 2006-November 2008 by the factor of 0.28. It means that the palm oil price over period November 2006-November 2008 is more volatile than over period January 2003-October 2006. Indeed, the price volatility fluctuates randomly across the time. We will extend the potential diffusion model to develop the volatility dynamics of the commodity price as proposed by Anderluch and Borovkova ([1]).

At first we consider the volatility parameter σ as the long-term average volatility. Let $\sigma(t)$ be the time-dependent volatility parameter. Then the potential diffusion model can be written as follow:

$$dX(t) = -U'(X(t))dt + \sigma(t)dW(t). \qquad (6)$$

Using the discrete time model of eq. (6) we can be defined the volatility on day i implied by the potential diffusion model by following equation

$$\sigma^2(t_i) = [X(t_{i+1}) - X(t_i) + U'(X(t_i))dt]^2. \qquad (7)$$

Recall that the invariant distribution of the process (3) is the Gibbs distribution. Hence, the potential $U(X(t))$ can be estimated, together with the volatility σ, from historical data by eq.(5). Then we can estimate by

$$\hat{\sigma}^2(t_i) = \left[X(t_i) - X(t_{i-1}) + \frac{\hat{\sigma}^2}{2}\hat{G}'_\sigma(X(t_{i-1}))dt\right]^2,$$

where $\hat{\sigma}$ and $\hat{G}_\sigma$ are respectively the estimates for the long-term volatility and the scaled potential. It is not difficult to modify such an approach to estimate the daily volatility in case we include trend in the log spot price model. The model-implied volatility $\sigma(t_i)$ can be used as a volatility measure instead of the realized volatility. We can use it for option pricing, risk management (e.g., VaR), portfolio management, etc.

Anderluch and Borovkova ([1]) explained that according to the discrete-time model (6), the magnitude of the daily log price increments is partly determined by the derivative of the potential and partly by random fluctuations. The presence of a deterministic component given by the potential’s derivative (and also trend if it occurs) is what distinguishes the volatility measure (7) from the realized volatility: the deterministic component plays a larger role if the current price is far from one of the attraction points. Near the attraction points the derivative of the potential is close to zero, so the random fluctuations are predominantly responsible for price movements.

In derivatives pricing, e.g., option pricing, the volatility is an important parameter which is often cannot be directly observed. The implied volatility, the volatility which is implied from the liquid option by inverting the option price formula such as the Black-Scholes formula, is usually used. Unfortunately, the commodity option markets are not as developed and liquid as the stock markets. Hence, the liquid option price data is often unavailable. In such a case, market participants will deal with the historical (realized) volatility or the GRACH volatility. For example, in order to value the option price we can use the average of the realized volatility over a certain period. The averaging period is usually equals to the time to the option maturity. For example, if the time to the option maturity is 1 month, we will use the historical data of the past 1 month to forecast the next-1 month average volatility.

We apply such an approach to estimate the daily volatility of the daily prices of the rolling gold and the palm oil. The daily volatility implied by the potential diffusion model is presented in Figure 7. We compare it to the realized volatility based on the historical data of the past 5, 20 and 60 days (equals to 1 week, 1 month and 3 months). Those figures show that the model-implied volatility is more erratic than the realized volatility, but both realized and model-implied volatility have the similar patterns.

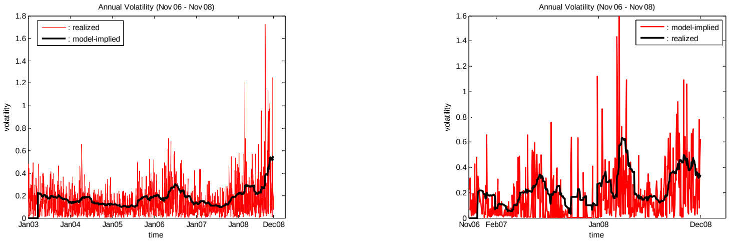


Figure 8. Model-implied volatility vs. realized volatility based on the historical data of the past 5, 20 and 60 days of the rolling gold (*left*) and the palm oil (*right*)

As we have mentioned that the model-implied volatility $\sigma(t,x)$ can be used as a volatility measure instead of the realized volatility. Here, we will compare the realized volatility based on the historical data of the past 5, 20 and 60 days to the average of the model-implied volatility of the past 5, 20 and 60 days, as did to the option whose the time to maturity is 1 week, 1 month and 3 months. The results given in Figures 9 and 11 show that the average of the model-implied volatility looks like the realized volatility. On average, the realized volatility is higher than the average of the model-implied volatility, but discrepancy of them is getting lower for the shorter averaging period of the model implied volatility. That feature is justified by the Q-Q plot given in Figures 10 and 12. Indeed, for the averaging period of the model-implied volatility is equals to 5 days (1 week trading day), the distribution of the realized volatility can approximate well the realized volatility as shown by Figures 10 (right) and 12 (right).

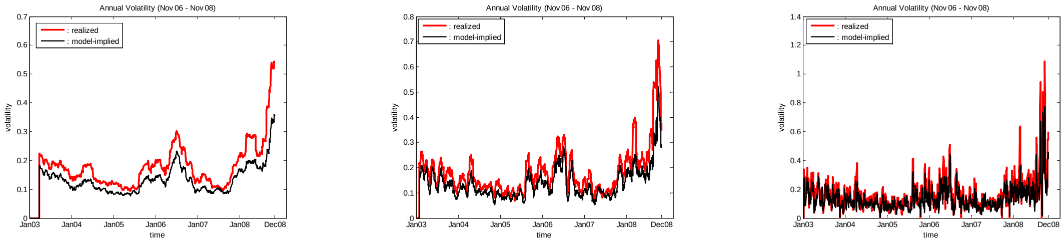


Figure 9. The Average of model-implied volatility vs. the realized volatility based on the historical data of the past 5, 20 and 60 days, the rolling gold

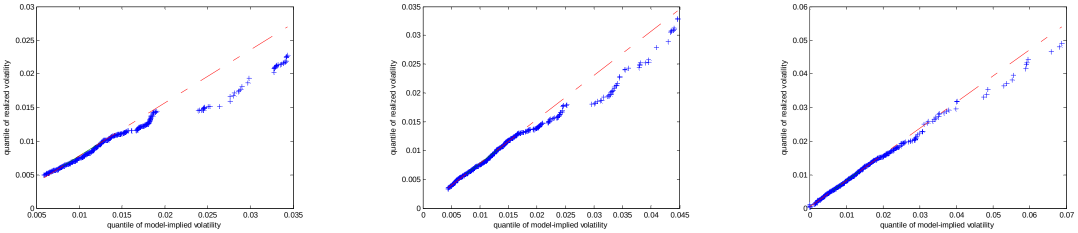


Figure 10. Q-Q plot of the average of the model-implied volatility vs. the realized volatility based on the historical data of the past 5, 20 and 60 days, the rolling gold

4 Conclusions and future work

In reality, it is often that the commodity price dynamics do not follow the GBM model. The potential diffusion model can be a better alternative model since such a model is in accordance with the market equilibrium setting. Moreover, the potential diffusion model can deal with multiple attraction regions, while the mean-reversion diffusion model can only deal with a single attraction region. Those make the potential diffusion a realistic model and it performs better than the mean-reversion diffusion model for modeling the commodity price dynamics. Such a model also performs well in the presence of trend in the price dynamics.

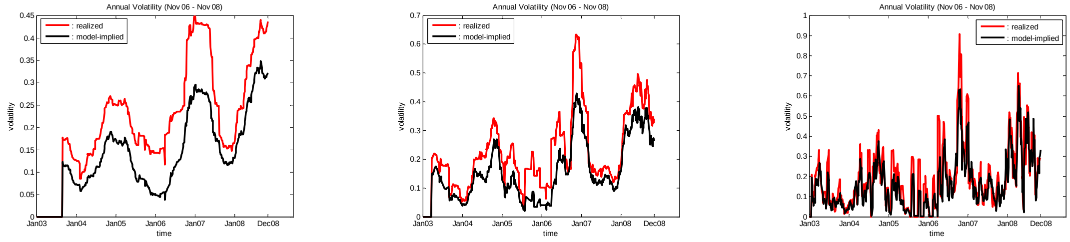


Figure 11. The average of model-implied volatility vs. the realized volatility based on the historical data of the past 5, 20 and 60 days, the palm oil

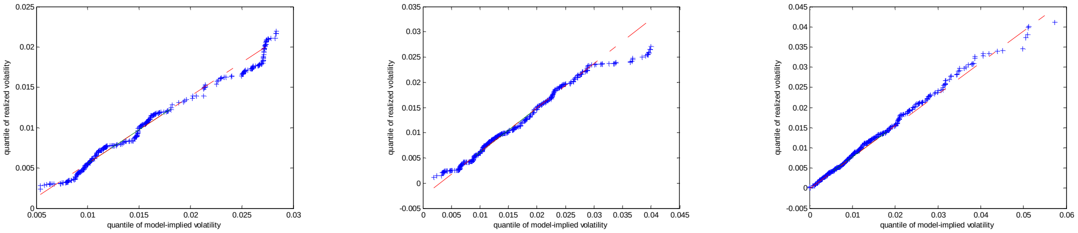


Figure 12. Q-Q plot of the average of the model-implied volatility vs. the realized volatility based on the historical data of the past 5, 20 and 60 days, the palm oil

We can extend the potential diffusion model to develop the price volatility dynamics. Simulation study shows that the model-implied volatility is higher and more erratic than the realized volatility but they have the same pattern. Indeed simulation study shows that the average of the model-implied volatility looks like the realized volatility. Hence, the model-implied volatility can be used as a volatility measure instead of the realized volatility.

Since the potential diffusion model can perform better than the GBM model to model the commodity price dynamics, it is interesting to develop the option price model based on the potential diffusion model. Here we will use the average of the model-implied volatility as input parameter of the option price model. We also intend to test performance of the model-implied volatility applied to the risk management such as Value-at-Risk.

5 References

[1] Anderluch, J. and S. Borovkova. 2008. Volatility modeling and commodity option pricing by the potential function approach. The *European Journal of Finance*, 14(2), 91-113.

[2] Bernard, J.T., L. Khalaf, M. Kichian and S. McMohan. 2006. Forecasting commodity prices: GARCH, jumps and mean reversion. *Bank of Canada Working Paper* 2006-14, April 2006.

[3] Black, F. 1976. The Pricing of commodity contracts. *Journal of Financial Economics* 3, 167-179.

[4] Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307-327.

[5] Borovkova, S., H.G. Dehling, J. Renkema and H. Tukelen. 2003. A potential approach to financial time series modelling. *Computational Economics*, 22, 139-161.

[6] Borovkova, S. and Permana, F.J. 2006. Modelling electricity prices by potential jump-diffusion. *Stochastic Finance*, 239-264.

[7] Borovkova, S. , Permana, F.J. and Pavlyukevich. 2009. Modelling electricity prices by potential Lévy diffusions (to appear in Journal of Energy Markets).

[8] Geman, H. 2005. *Commodities and Commodity Derivatives: Modeling and Pricing for Agricultural, Metal and Energy*. Wiley Finance.

[9] Gibson and Schwartz .1990. Stochastic convenience yield and the pricing of oil contingent claims. *Journal of Finance*, 45(3), 959-976.

[10] Hafner, C. 2003. Simple approximation for option pricing under mean reversion and stochastic volatility. *Econometric Institute Report* EI 2003-10, Erasmus University, Rotterdam.

[11] Huisman, R. and R. Mahieu. 2001. Regime jumps in electricity markets. *Energy and Power Risk Managements*. Risk Publications.

[12] Lucia, J. and E. S. Schwartz. 2002. Electricity prices and power derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research* 5(1), 5-50.

[13] Matkovsky, B.J. and Z. Schuss. 1981. Eigenvalues of the Fokker-Planck operator and the approach to equilibrium for diffusions in potential fields. *SIAM Journal of Finance* 33 (1), 177-186.

[14] Nochai, R. and T. Nochai. 2006. ARIMA model for forecasting oil palm price. *Proceedings of the 2nd IMT-GT Regional Conference on Mathematics, Statistics and Applications*, University Sains Malaysia, June 13-15, 2006.

[15] Permana, F.J., Lesmono, D. and Chendra E. (2009a). Palm oil price model of Indonesia market. Submitted to *Proceedings of the 5th Asian Mathematical Conference (ACM)*, June 22-26 2009, Kuala Lumpur, Malaysia.

[16] Permana, F.J., Lesmono, D. and Chendra E. (2009a). Submitted to *Proceedings of the 4th International Conference on Research and Education in Mathematics (ICREM)*, October 21-23, 2009, Kuala Lumpur, Malaysia.

[17] Pilipovic, D. (1998). Valuing and Managing Energy Derivatives. McGraw Hill Publications.

[18] Schwartz, E. S. 1997. The stochastic behaviour of commodity prices: implication for valuation and hedging. The *Journal of Finance*, 52(3), 923-973.

[19] Sørensen, C. Seasonality in Agricultural Futures. http://www.cbs.dk/departments/finance/vip/carsten/index.html.