Proceedings of the 4th International Conference on Research and Education in Mathematics

"Meeting Challenges of Global Research and Education in Mathematical Sciences"

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21st - 23rd October 2009

Jointly Organized by :

Institute for Mathematical Research, Universiti Putra Malaysia (INSPEM)

Institute of Mathematics, Vietnam Academy of Science & Technology (IMVAST)

Faculty of Mathematics & Natural Sciences, Bandung Institute of Technology, Indonesia (ITB)

Malaysian Mathematical Sciences Society (PERSAMA)

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Editors

Mohamad Rushdan Md. Said, Hishamuddin Zainuddin, Noor Akma Ibrahim, Rohani Ahmad Tarmizi and Habshah Midi

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PREFACE

The International Conference on Research and Education in Mathematics (ICREM) is a biennial conference organized by the Institute for Mathematical Research, Universiti Putra Malaysia. The fourth conference (ICREM4) differs from the earlier ones by having joint organizers from abroad namely Institute of Mathematics, Vietnam Academy of Science and Technology and the faculty of mathematics and natural Sciences, Bandung Institute of Technology, Indonesia as well as local ones i.e. Malaysian Society Mathematical Sciences and Malaysian Society for Cryptology Research. The conference is also supported by Abdus Salam International Centre for Theoretical Physics, Trieste, Italy and United Nations Educational, Scientific and Cultural Organization (UNESCO).

The present proceedings capture part of the excitement of the conference documenting well over one hundred papers contributed by participants from more than twenty countries. They cover all four main areas in mathematical sciences i.e. pure mathematics, Applied Mathematics and Theoretical Sciences, Statistics and Mathematics Education but ones that mirror the interests of the regional community of mathematical scientist and practitioners.

The Organizers would like to thank all the invited speakers and participants for their contributions in making this conference a success and hence brought forth this valuable proceedings.

We would like to express our deepest appreciation to all sponsors of the conference, without which the conference may not be realized. Specifically, we would like to mention Abdus Salam International Centre for Theoretical Physics UNESCO, Ministry of Higher Education and Ministry of Science, Technology and Innovations, Malaysia. Last, but not least, our utmost thanks to the management of the University for their unfailing support towards our activities.

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STOCHASTIC PRICE PROCESS MODELS OF ROLLING GOLD TRADED IN INDONESIA MARKET

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Abstract. *Modeling commodity price process represented by a stochastic differential equation is essential for developing the risk managent tools, e.g., options, besides for forecasting future prices. However, gold has spesific features compared to other commodities such as agricultural and energy commodies, even compared to other metals. Gold is the most popular commodity used as an investment. Hence, gold is often used to hedge the risks against economic, political, social or currency-based crisis. We model the rolling gold spot prices traded in Indonesia market as commonly applied to model the commodity price, that is as the the sum of the deterministic and stochastic components. Our investigation shows that the gold price process does not exhibit seasonality but presents trend influenced by inflation. That feature is captured as the deterministic component model. To describe the stochastic component, we investigate performances of three models: Geometric Brownian Motion, mean-reversion diffusion and potential diffusion models. Performances of those three models are measured by comparing the distributional characteristics obtained from the original data and from those models to find the most suitable model for rolling gold prices. Then, based on the most suitable model, we forecast future spot rolling gold prices and apply some statistical tests to investigate performances of the model.*

Key words: Geometric Brownian motion, mean-reversion diffusion, potential diffusion model, rolling gold prices, Indonesia market.

1 Introduction

Gold has been used as an investment for a long time along with other precious metals such as copper, silver, tin, lead, alluminium, zinc, nickel, palladium and platinum. However, gold has become the most popular metals used as investment compared to other abovementioned metals. People invest in gold to hedge or protect against social, economic or political crises such as war, market declines, inflation or political uncertainty in the country of in the world. People also note that gold prices have an increasing trend over the last decade; so this is also another reason for investing in gold. Gold also used in industry for art and jewellery and in the past it was used as money where all other commodities' prices were measured in gold price.

In this paper, we model the rolling gold spot prices traded in Indonesia market by considering deterministic and stochastic components of the prices. Most researchers and practitioners in commodity markets have proposed and used this approach in modelling commodity prices (e.g. Pilipovic (1998), Huisman and Mahieu (2001), Lucia and Schwartz (2002) and Sorensen (2002)). The deterministic component includes the behaviour of trend and seasonality of the gold spot prices, while for the stochastic component we will consider three models to describe it, viz. Geometric Brownian Motion (GBM), mean-reversion diffusion and potential diffusion. The main purpose of this paper is to find the appropriate model for describing the behaviour of the rolling gold prices traded in Indonesia market. We collected the rolling gold prices, quoted in Indonesian Rupiah (IDR)/grams, traded in the Jakarta Futures Exchange from January 2003 to November 2008. The prices are based on the gold physical market Loco London and are collected from Reuters at the end of trading day. The prices then are converted into IDR using the exchange rates from Reuters.

The rest of this paper is organized as follows. In the next chapter, we discuss the stochastic models we used in modelling rolling gold prices: GBM, mean-reversion diffusion and potential diffusion. Section 3 is dealing with modelling the dynamics of the rolling gold prices. Performance measure of each model is based on the comparison of the first four moments of the generated paths to the original rolling gold prices, as proposed by Geman (2005). Based on the best model, we then attempt to investigate its performance in forecasting the future pices for the next 5, 20 and 60 days. Conclusions and further research are discussed in the last section.

2 The Deterministic and Stochastic Models

The main purpose of this paper is to find a suitable model for the rolling gold spot prices by considering deterministic and stochastic components. For the deterministic component, we investigate the possibility of the occurrence of trend and seasonality in the data while in the stochastic components, we propose to apply three stochastic models: geometric Brownian motion, mean-reversion diffusion and potential diffusion models. Makridakis, Wheelwright and Hyndman (1998) describe a seasonal pattern when the data is influenced by the seasonal factors such as the quarter of the year, the month or day of the week. A seasonal pattern repeats itself over fixed intervals of time. A trend pattern exists when the data exhibit a long-term increase or decrease. The occurence of seasonality in the data can be identified when there is a large autocorrelation coefficient or partial autocorrelation coefficient at the seasonal lag.

The Geometric Brownian Motion (GBM) model was first used to model the stock price. Black and Scholes (1973) introduced the Black-Scholes model for option price valuation by assuming that the stock price follows a GBM and from there the GBM becomes popular and widely used for asset price model. Black (1976) used the GBM to model the futures commodity price dynamic. The asset price process following the GBM can be represented by the following stochastic differential equation (SDE):

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t)$$

where $S(t)$: the asset price, e.g., the spot price of commodity, at time t, μ: the expected return of asset, σ: the volatility and $dW(t)$: the increment of Wiener process. By setting $G(S(t),t) = log(S(t))$ and applying the Ito's Lemma, the process of $S(t)$ will follow the SDE

$$d\big(log(S(t))\big) = \left(\mu - \tfrac{1}{2}\sigma^2\right)dt + \sigma dW(t) \qquad (1)$$

Equation (1) implies that the log return, $log\left(\frac{S(t+dt)}{S(t)}\right)$, is normally distributed with:

Mean $\quad E\left[log\left(\frac{S(t+dt)}{S(t)}\right)\right] = (\mu - \frac{1}{2}\sigma^2)dt$

Variance $\quad Var\left[log\left(\frac{S(t+dt)}{S(t)}\right)\right] = \sigma^2 dt.$

The unexpected events, e.g., the flood, the hurricane or the war, can cause the harvest failure on commodities and imply a shortage. In such situations, the price will increase because supply and demand are not balanced. In equilibrium setting, the price will eventually return towards the mean level after the event goes away and the supply and demand are balanced. On the other hand, overproduction will cause the price goes down. Again, in the equilibrium setting, the price will return towards the mean level after the producers decrease the production and then the supply and the demand are balanced. The commodity spot price modeling using a mean-reversion model is more realistic than the GBM model since the mean-reversion model can accommodate such situations. Using the GBM model, those unexpected events will always be considered as a normal event without consideration the prior price level and the probability of returning to the mean level.

The mean-reversion diffusion model is first introduced by Vasicek to model the random evolution of interest rates. Nowadays, such a model is widely incorporated in energy (Borovkova and Permana (2006), Borovkova, Permana and Pavlyukevich (2009), Lucia and Schwartz (2002)), commodity price such as agricultural product, gold, copper (Bernard, et.al (2006), Schwartz (1997), Pilipovic (1998), Pindyck and Rubinfeld (1991)) and interest rate (Cox, Ingersoll and Ross (1985)). The asset price process following the mean-reversion diffusion model can be represented by the following SDE:

$$d\Big(log\big(S(t)\big)\Big) = \alpha\Big(m - log\big(S(t)\big)\Big)dt + \sigma dW(t) \qquad (2)$$

whereα: the mean-reversion rate, m: the mean-reversion value and $dW(t)$: the increment of Wiener process.

Price clustering on commodity prices tends to concentrate in a number of attraction regions. It means that the price moves among these attraction regions, although the time spent at a given region cannot be predicted and can be long.

Mean-reversion diffusion model is more realistic in terms of the equilibrium setting, although it also has a limitation. Suppose that an unexpected event causes the price leaves the mean level. After that event goes away, the supply and demand are balanced, the price will then return towards the mean level. If the current mean level is the same as the previous mean levels, a mean-reversion diffusion model can still be an appropriate model for the price dynamic. Unfortunately, the current level sometimes is different from the previous mean levels. In that case, the price dynamic will have multiple attraction regions and a mean-reversion diffusion model cannot deal with such a situation. Here, the potential diffusion model will be more appropriate than the mean-reversion diffusion model.

The commodity price modeling using a potential diffusion model has been introduced by Borovkova et. al. (2003). The price process following a potential diffusion model is represented by the SDE:

$$dX(t) = -U'\big(X(t)\big)dt + \sigma dW(t), \qquad (3)$$

where $X(t) = log(S(t))$, $U: R \to R$ is a twice continuously differentiable function such that $U(t) \to \infty$ as $[x] \to \infty$ and $\int_{-\infty}^{\infty} \exp\left(-\frac{2U(X(t))}{\sigma^2}\right) dx < \infty$. Those conditions assure that the invariant distribution of the process $(X(t))$ is a Gibbs distribution with density

$$\pi_\sigma(x) = exp\left(-\frac{2U(x)}{\sigma^2}\right) \quad (4)$$

(for proof see e.g., [12]).
Equation (4) gives a one-to-one correspondence between the invariant distribution of the process and the diffusion's drift, given by the potential.

The potential $U(X(t))$ can be estimated, together with the volatility σ, from historical data by first estimating

$$G_\sigma(x) = \frac{2}{\sigma^2} U(x) = -log(\pi_\sigma(x))$$

by

$$\hat{G}_\sigma(x) = -log(\hat{\pi}(x)),$$

where $\hat{\pi}$ is some estimate of the observations' marginal density, e.g., a kernel density estimator or a histogram smoothed by a polynomial or a sum of Gaussian densities.

The potential diffusion model has some advantages. It is realistic since it is close in the spirit to a mean-reversion. It allows the data to have the multiple attraction regions, while a mean-reversion diffusion model can only deal with a single attraction region. A non-constant reversal rate in the potential diffusion model is also permitted. Indeed, the reversal rate incorporated in the potential diffusion model is a continuous function of a distance to the mean price level. Hence, the mean-reversion diffusion model can be considered as a special case of the potential diffusion model with a constant of reversal rate since its potential function is represented by a quadratic function.

The modeling procedure is starting by making discretization of the SDEs (1), (2) and (3) and applying the Euler Scheme. Discretization of SDEs (1), (2) and (3) yields the following equations:

$$log(S(t_{i+1})) - log(S(t_i)) = \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\,\epsilon_t,$$

$$log(S(t_{i+1})) - log(S(t_i)) = \alpha\left(m - log(S(t_i))\right)\Delta t + \sigma\sqrt{\Delta t}\,\epsilon_t,$$

$$X(t_{i+1}) - X(t_i) = -U'(X(t_i))\Delta t + \sigma\sqrt{\Delta t}\epsilon_t\,,$$

where ϵ_t: standard normal random variable, $X(t_i) = log(S(t_i))$ and Δt: the unit of time step.
Choosing the appropriate model yields the parameter estimation tasks. In this paper, the parameter estimates will be obtained by applying least squares or maximum likelihood method.

3 Modelling Rolling Gold Prices

We work on the rolling gold spot prices traded in Indonesia market over the period of January 2003 – November 2008. Figure 1 and 2 below represent the rolling gold spot prices and log spot prices over that period. We can see clearly that the data exhibit trend, that is the spot prices are increasing over time, but for the seasonality we have calculated the autocorrelation and partial autocorrelation and found that there is no evidence of seasonality on the data. Therefore, for our modelling purpose, we only include the trend as the deterministic component. In Figure 2, we include the trend line on the rolling gold log spot prices and we found the trend equation has slope of 11,3921 and intercept of 0,007 with coefficient determination of 96,43%.

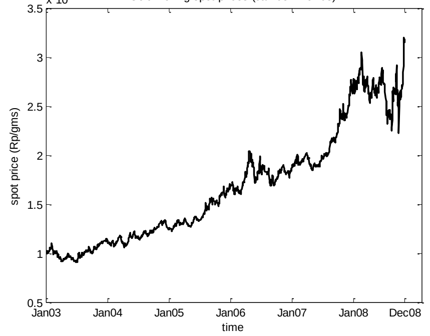

Figure 1. Rolling Gold spot price
(January 2003 – November 2008)

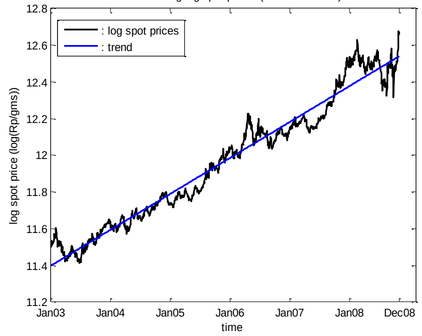

Figure 2. Rolling Gold log spot price
(January 2003 – November 2008)

The next step is modelling the stochastic components using three models, namely the GBM, mean-reversion diffusion and potential diffusion. First we find the parameter estimates for each model, by discretize the corresponding SDEs to each model and using the least square method. In this procedure we also include the deterministic component found in the previous step. For the potential diffusion, we use 6th degree polynomial. Tabel 1 gives the parameter estimates for each model with annual volatilities are given in bracket in each model. Based on these parameters, we generate 1000 paths of the rolling gold log spot prices and results are given in Table 2.

Table 1. Parameter estimates for rolling gold prices (January 2003 – November 2008)

GBM		Mean-reversion diffusion			Potential diffusion
μ	σ	α	m	σ	σ
0.0001	0.0131 (20.71%)	-0.021	0.0005	0.013 (20.55%)	0.0139 (21.98%)

Table 2 gives comparison of the first four moments of the generated paths for each models and the original log prices. From that table, we can see that all the models can fit the mean and standard deviation quite good. But the GBM fail to fit the skewness, giving a negative skewness. Also for the kurtosis, both mean-reversion diffusion and potential diffusion give a quite good matching campared to the GBM. Potential diffusion with 6th degree polynomial performs a slightly better than the mean-reversion diffussion in fitting the skewness, giving value that closer to the original log prices skewness. Due to this fact we conclude that potential diffusion with 6th degree polynomial along with the trend for the deterministic component is a slightly better model compared with other models. Figures 3, 4 and 5 give the 6th degree degree polynomial potential along with their rate of reversion and one of generated log prices path.

Table 2. The first four moments of the original and generated rolling gold log prices (January 2003 – November 2008)

Moment	Original log prices	Generated log prices		
		GBM	Mean-reversion diffusion	Potential diffusion
Mean	11.9684	12.0733	11.972	11.9716
Std. deviation	0.3364	0.3646	0.3307	0.3295
Skewness	0.1726	-0.0132	0.0441	0.0544
Kurtosis	1.8622	2.1688	1.8249	1.8215

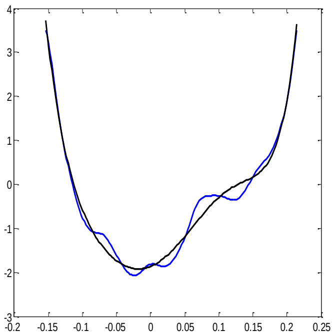

Figure 3. The 6th degree polynomial Potential (Jan 2003 – Nov 2008)

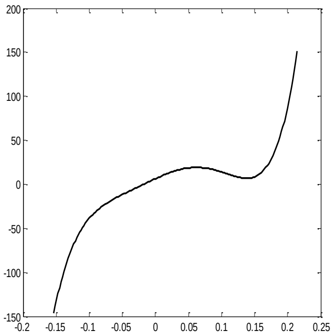

Figure 4. Rate of reversion (Jan 2003 – Nov 2008)

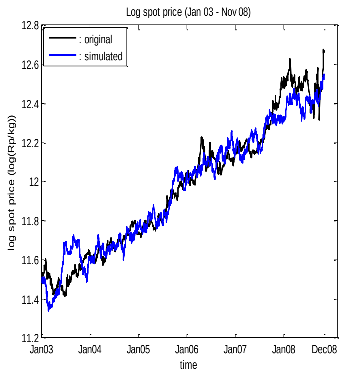

Figure 5. Generated log prices (Jan 2003 – Nov 2008)

Many papers propose time series models to predict the future spot prices and they perform quite well. In this paper, we try to investigate the performance of the linear trend plus the 6th degree polynomial potential model in predicting the future rolling gold spot prices. Based on the current rolling gold spot prices and the parameter estimates we found for the suitable model, 1000 rolling gold prices representing price forecast for the next 5, 20 and 60 days (the next 1-week, the next 1-month and the next 3-month) are generated. The distributional characteristics of relative errors are calculated and given in Table 3 along with 95% confidence interval for the price forecast in Figure 6. We define the relative errors as the discrepancy between the original prices obtained from the market and the forecasted prices from the Monte Carlo simulation relative to the original prices. From Table 3 we found that the means of relative errors are all positive for n=5, 20 and 60. They indicate that forecasted prices are overestimated. Also from Figure 6, we can see that the original log spot prices always lie within the 95% confidence interval for n=5, 20 and 60 days. However, forecasted prices for the next 1-week give smallest standard error compared with the forecasted prices for the next 1-month and the next 3-month.

This results are inline with a common philosophy in forecasting that is “use as many as historical data and use them to forecast values that are not far away into the future”. In general, our best model, trend plus 6th degree polynomial potential perform quite well in term of predicting the future log spot prices.

Table 3. Distributional characteristics of relative error of the n-day forecast log prices using 6th degree polynomial potential

Distributional characteristic	The next n-day forecast		
	n=60	n=20	n=5
Mean	0.0489	0.0146	0.0002
Standard error	0.0349	0.0215	0.011
Minimum	-3.5069	-2.3886	-1.3241
Maximum	3.635	2.45	1.4308

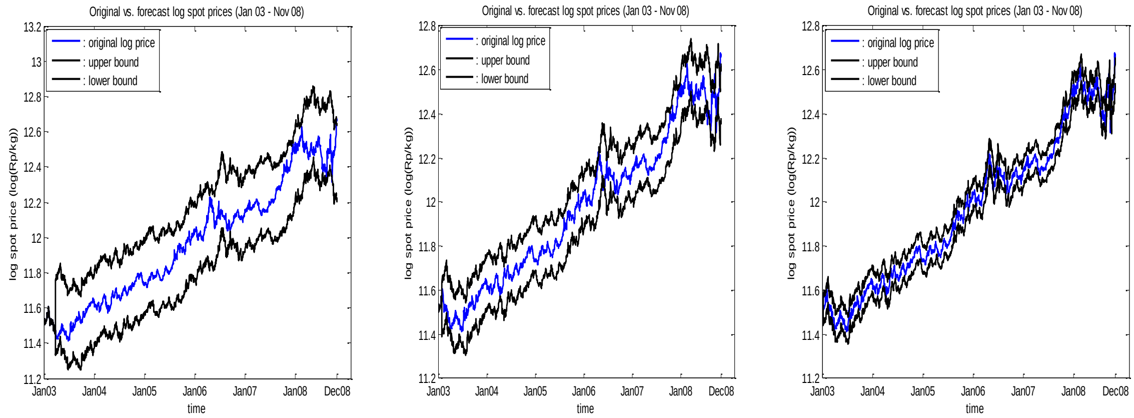

Figure 6. The 95% confidence interval of the n-day forcast log price, potential diffusion 6th degree model, January 2003 – November 2008 n=60 (left), n=20 (centre), n=5 (right)

4. Conclusions and Further Research

We have modelled the rolling gold prices traded in Indonesia market as a combination of deterministic and stochastic component over the period of January 2003 to November 2008. Our investigation concludes that for the deterministic component, rolling gold prices do not exhibit seasonality but they do exhibit trend. For the stochastic component, we found that potential diffusion with 6th degree polynomial potential performs a slightly better than the GBM and mean-reversion diffusion since it gives parameters’ values, such as the mean, standard deviation, skewness and kurtosis that are closer to the parameters of the original log rolling gold prices.

In this paper, we use the stochastic models for describing the dynamics of rolling gold prices as a stepping stone for our further research that is to develop models for option pricing on the rolling gold, as proposed by Anderluch and Borovkova (2008). This further research can not be done if we only include the deterministic component, although as mentioned before that the trend gives 96,43% coefficient of determination, a quite good representation for the original rolling gold prices. Another avenue for further research is to develop a volatility model for the dynamic of the rolling gold prices, as proposed by Heston (1993) for bond and currency options.. In this paper we assume that the volatility of the rolling gold prices is constant although it is not quite appropriate in reality. Therefore, developing a volatility model is an essensial way in order to better describe the dynamics of the rolling gold prices.

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