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An Application of Linear Programming in Grass Supply at Dutch Zoos

Abstract

This report is dealing with the problem of Zooland, a company that specializes in the supply of animal feed to zoos. A weekly delivery schedule of savannah grass and dutch grass from Zooland depots to the three important Dutch zoos will be arranged such that the schedule can both maximize Zooland's profit and fulfill the wishes of the hippopotami and the elephants, even when there is a constraint.

Intisari

Laporan ini menguraikan masalah yang dihadapi oleh Zooland, suatu perusahaan yang mengkhususkan diri dalam menyediakan makanan ternak ke kebun binatang-kebun binatang. Jadwal mingguan untuk pengiriman rumput savanah dan rumput belanda dari gudang Zooland ke tiga kebun binatang penting di Belanda akan diatur sehingga jadwal tersebut dapat memaksimalkan keuntungan Zooland dan memenuhi harapan kuda nil-kuda nil dan gajah-gajah, walaupun ada batasan.

I. Introduction

Zooland is a company with a quite unusual type of clients. Its clients are elephants and hippopotami. Zooland specializes in the supply of animal feed to zoos. Zooland is the supplier of three important Dutch zoos : the Noorder Dierenpark in Emmen, Artis in Amsterdam (the oldest zoo of the Netherlands) and Safaripark Beekse Bergen. Table 1.1. shows how many elephants and hippopotami these three zoos possess.

Zooland has two depots in the Netherlands. In these depots enormous quantities of grass are kept in stock. Zooland is the supplier of two different types of grass : the quite expensive, imported Savannah grass and the cheaper Dutch grass.

Hippopotami are, apart from dangerous also very big eaters. After having spent the whole day in the water, a hippopotamus eats every evening about 80 kilograms of grass a day. Elephants eat even more, each elephant eats about 120 kilograms of grass a day.

Table 1.1. The number of elephants and hippopotami in the different zoos

Zoo	Ele-phants	Hippo-potami
Artis Amsterdam (AA)	8	4
Noorder Dierenpark Emmen (NDE)	10	10
Safaripark Beekse Bergen (SBB)	15	6

Although both animals have a clear preference for savannah grass, in the zoos they eat a mix of both types of grass. A hippopotamus receives a mix of at least 30% of savannah grass. Elephants are more difficult to fool; they get at least 50% of savannah grass.

Zooland delivers grass to the zoos weekly. In table 1.2. the selling price and the purchase price of the types of grass is shown. (The purchasing price is the price that Zooland has to pay for buying the grass).

Table 1.2. The purchasing and selling prices per 1000 kilograms of grass

	Selling price	Purchasing price
Dutch grass	fl. 190	fl. 69,70
Savannah grass	fl. 250	fl.182,30

Zooland depends on suppliers in the Netherlands and abroad. Therefore, the Zooland depots have a restricted delivery capacity per week. These capacities are given in table 1.3.

The transport costs of the grass are assumed to be proportional to the quantity of grass and the distance in kilometers in travel. The cost of transporting 1000 kilograms of grass are fl. 0,35 per kilometer. The distances between the zoos and the depots are found in table 1.4.

Table 1.3. The weekly delivery capacities of grass of the depots of Zooland (in 1000 kgs)

	Savannah grass	Dutch grass
Zooland Rotterdam	12	10
Zooland Utrecht	6	20

Table 1.4 Distances in km

	AA	NDE	SBB
Zooland Rotterdam	70	210	81
Zooland Utrecht	33	162	70

We will design a delivery schedule which will maximize Zooland's profit and also fulfill the zoo's need.

II. Processing of Data

We first make some further computation based on the given data that can make us find the solution more easily. The results are given in the following tables :

Table 2.1. The number of grass needed in a week (in 1000 kgs)

ZOO	Ele-phant	Hippo-potami	Total	Min Savan-nah #)	Max Dutch ##)
AA	6.72 *)	2.24 **)	8.96	4.03	4.93
NDE	8.40	5.60	14.00	5.88	8.12
SBB	12.60	3.36	15.96	7.31	8.65

*) = 8 elephants x 120 kgs / elephant x 7 days
**) = 4 hippopotami x 80 kgs / hippopotamus x 7 days.
#) = 50% x (*) + 30 % x (**)
##) = 50% x (*) + 70 % x (**)

Table 2.3 Revenue per 1000 kgs of grass

	Selling price	Purchasing price	Revenue
Dutch grass	fl. 190	fl. 69.70	fl.120.30
Savannah grass	fl. 250	fl. 182.30	fl. 67.70

Table 2.3 The transportation cost per 1000 kgs from depots of Zooland to the zoos

	AA	NDE	SBB
Zooland Rotter-dam	fl. 24.50 (= 70 km x fl. 0.35/km)	fl. 73.50	fl.28.35
Zooland Utrecht	fl. 11.55	fl. 56.70	fl. 24.50

III. Model Formulation

The model of Zooland's problem can be described as (see the following figure) a transportation problem.

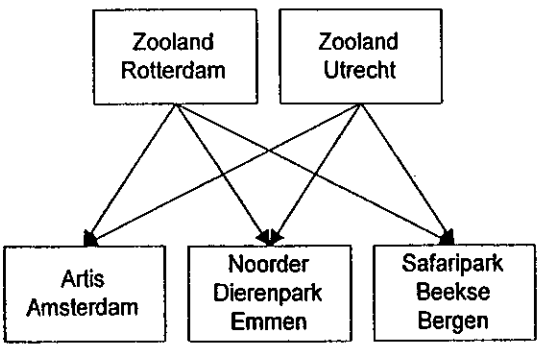

Figure 3.1 Representation of the problem

The Decision Variables will be : [1,2]

X_{ijk} ; $i = 1, 2$ $j = 1, 2, 3$ $k = 1, 2$

$i = 1$: Zooland Rotterdam

$i = 2$: Zooland Utrecht
$j = 1$: Artis Amsterdam
$j = 2$: Noorder Dierenpark Emmen
$j = 3$: Safaripark Beekse Bergen
$k = 1$: Savannah grass
$k = 2$: Dutch grass

X_{ijk} defines the amount of grass k (in 1000 kgs) transported from depots i to zoo j.

Constraints are :

a) Demand per week

- Artis Amsterdam
  $X_{111} + X_{211} \geq 4.032$
  $X_{112} + X_{212} \leq 4.928$
  $\sum_{i=1}^{2} \sum_{k=1}^{2} X_{i1k} = 8.96$

- Noorder Dierenpark Emmen
  $X_{121} + X_{221} \geq 5.88$
  $X_{122} + X_{222} \leq 8.12$
  $\sum_{i=1}^{2} \sum_{k=1}^{2} X_{i2k} = 14$

- Safaripark Beekse Bergen
  $X_{131} + X_{231} \geq 7.308$
  $X_{132} + X_{232} \leq 8.652$
  $\sum_{i=1}^{2} \sum_{k=1}^{2} X_{i3k} = 15.96$

b) Depot's capacity

	Savannah	Dutch
Zooland Rotterdam	$\sum_{j=1}^{3} X_{1j_1} \leq 12$	$\sum_{j=1}^{3} X_{1j_2} \leq 10$
Zooland Utrecht	$\sum_{j=1}^{3} X_{2j_1} \leq 6$	$\sum_{j=1}^{3} X_{2j_2} \leq 20$

c) Nonnegativeness :

$$X_{ijk} \geq 0$$

The objective function to be maximized is :

$$\textit{Profit} = \textit{Revenue} - \textit{Cost}$$

$$\textit{Revenue} = 120.0 \sum_{i=1}^{2}\sum_{j=1}^{3} X_{ij2} + 67.7 \sum_{i=1}^{2}\sum_{j=1}^{3} X_{ij1}$$

$$\textit{Cost} = 24.5 \sum_{k=1}^{2} X_{11k} + 73.5 \sum_{k=1}^{2} X_{12k} + 28.35 \sum_{k=1}^{2} X_{13k} + 11.55 \sum_{k=1}^{2} X_{21k} + 56.7 \sum_{k=1}^{2} X_{22k} + 24.5 \sum_{k=1}^{2} X_{23k}$$

Using the CPLEX software the model formulation becomes : [3]

```
\Problem name: zoolandlrev.lp

Maximize
 obj: 95.8 x112 + 46.8 x122 + 108.75 x212 + 63.6 x222 + 43.2 x111 - 5.8
x121
 + 56.15 x211 + 11 x221 + 39.35 x131 + 91.95 x132 + 43.2 x231 + 95.8
x232
Subject To
 c1:  x111 + x211 >= 4.032
 c2:  x112 + x212 <= 4.928
 c3:  x121 + x221 >= 5.88
 c4:  x122 + x222 <= 8.12
 c5:  x131 + x231 >= 7.308
 c6:  x132 + x232 <= 8.652
 c7:  x112 + x212 + x111 + x211  = 8.96
 c8:  x122 + x222 + x121 + x221  = 14
 c9:  x131 + x132 + x231 + x232  = 15.96
 c10: x111 + x121 + x131 <= 12
 c11: x211 + x221 + x231 <= 6
 c12: x112 + x122 + x132 <= 10
 c13: x212 + x222 + x232 <= 20
End
```

Table 4.1 Optimal Weekly Delivery Schedule for Zooland (in 1000 kgs)

Depot	Grass	Artis Amsterdam	Noorder Dierenpark Emmen	Safaripark Beekse Bergen	Total
Z. Rotterdam	Savannah	3.912	0	7.308	11.22
	Dutch	0	0	1.7	1.7
Z. Utrecht	Savannah	0.12	5.88	0	6
	Dutch	4.928	8.12	6.952	20
Total	Savannah	4.032	5.88	7.308	
	Dutch	4.928	8.12	8.642	

IV. Solution

The solution of Zooland's proble given by CPLEX is summarized in the table 4.1.

From table 4.1 we know that to maximize profits, Zooland has to follow the following schedule :

From Zooland Rotterdam :

- Deliver weekly, 3912 kgs savannah grass to Artis Amsterdam zoo and 7308 kgs of savannah grass to Safaripark Beekse Bergen zoo.
- Deliver weekly, 1700 kgs dutch grass only to Safaripark Beekse Bergen zoo

From Zooland Utrecht :

- Deliver weekly, 120 kgs savannah grass to Artis Amsterdam zoo and 5880 kgs of savannah grass to Noorder Dierenpark Emmen zoo.
- Deliver weekly, 4928 kgs dutch grass to Artis Amsterdam zoo, 8120 kgs dutch grass to Noorder Dierenpark Emmen zoo, and 6952 kgs dutch grass to Safaripark Beekse Bergen zoo

The optimal weekly profit will be fl 2402.66.

V. Development of the Problem

There are some problems with the roof of the Zooland depot in Utrecht. It needs to be repaired. During this reparation, the savannah grass can not be stocked in Zooland Utrecht. Therefore, Zooland rents an extra depot in Rotterdam just for the Savannah grass. This rented depot is quite small, it has a capacity of only 5500 kilograms. (Hence, during the time of reparation the suppliers of Zooland can supply less grass)

Based on these problems, to formulate the new model, we define a new variable X_{3j1} j = 1, 2, 3 and delete the variable X_{2j1} j = 1, 2, 3 . We define new constraints and objective function content.

The constraints are :

a) Demand per week

- Artis Amsterdam
  $X_{111} + X_{311} \geq 4.032$
  $X_{112} + X_{212} \leq 4.928$
  $X_{112} + X_{311} + X_{112} + X_{212} = 8.96$

- Noorder Dierenpark Emmen
  $X_{121} + X_{221} \geq 5.88$
  $X_{122} + X_{222} \leq 8.12$
  $X_{121} + X_{321} + X_{122} + X_{222} = 14$

- Safaripark Beekse Bergen
  $X_{131} + X_{231} \geq 7.308$
  $X_{132} + X_{232} \leq 8.652$
  $X_{131} + X_{331} + X_{132} + X_{232} = 15.96$

b) Depot's capacity

	Savannah	Dutch
Zooland Rotterdam	$\sum_{j=1}^{3} X_{1j_1} \leq 12$	$\sum_{j=1}^{3} X_{1j_2} \leq 10$
Zooland Utrecht		$\sum_{j=1}^{3} X_{2j_2} \leq 20$
Extra Depot Rotterdam	$\sum_{j=1}^{3} X_{3j_1} \leq 5.5$	

c) Nonnegativeness :

$$X_{ijk} \geq 0$$

i = 1, 2, 3; j = 1, 2, 3; k = 1, 2;
not included X_{2j1} $j = 1, 2, 3$

The objective function to be maximized is : ***Profit*** = ***Revenue - Cost***

but with an adjusted formulation :

Revenue =

$$120.3\,(X_{112} + X_{122} + X_{132} + X_{212} + X_{222} + X_{232}) + 67.7(X_{112} + X_{122} + X_{132} + X_{212} + X_{222} + X_{232})$$

Cost =

$$24.5 \sum_{k=1}^{2} X_{11k} + 73.5 \sum_{k=1}^{2} X_{12k} + 28.35 \sum_{k=1}^{2} X_{13k} + 11.55\, X_{212} + 56.7\, X_{222} + 24.5\, X_{232}$$

$$+ 24.5\, X_{311k} + 73.5\, X_{321} + 28.35\, X_{331}.$$

Using CPLEX, the new model formulation becomes :

```
\Problem name: zooland2rev.lp

Maximize
 obj: 95.8 x112 + 46.8 x122 + 108.75 x212 + 63.6 x222 + 43.2 x111 - 5.8
x121
 + 43.2 x311 - 5.8 x321 + 39.35 x131 + 91.95 x132 + 39.35 x331 + 95.8
x232
Subject To
 c1:  x111 + x311 >= 4.032
 c2:  x112 + x212 <= 4.928
 c3:  x121 + x321 >= 5.88
 c4:  x122 + x222 <= 8.12
 c5:  x131 + x331 >= 7.308
 c6:  x132 + x232 <= 8.652
 c7:  x112 + x212 + x111 + x311  = 8.96
 c8:  x122 + x222 + x121 + x321  = 14
 c9:  x131 + x132 + x331 + x232  = 15.96
 c10: x111 + x121 + x131 <= 12
 c11: x311 + x321 + x331 <= 5.5
 c12: x112 + x122 + x132 <= 10
 c13: x212 + x222 + x232 <= 20
End
```

The solution of the new Zooland problem given by CPLEX is summarized in the following table:

Table 5 Optimal Weekly Delivery Schedule for Zooland (in 1000 kgs)

DEPOT	Grass	Artis Amsterdam	Noorder Dierenpark Emmen	Safaripark Beekse Bergen	Total
Z. Rotterdam	Savannah	4.032	5.88	1.808	11.72
	Dutch	0	0	1.7	1.7
Z. Utrecht	Dutch	4.928	8.12	6.952	20
New Depot	Savannah	0	0	5.5	5.5
Total	Savannah	4.032	5.88	7.308	
	Dutch	4.928	8.12	8.652	

Based on table 5 caused by the problem with the Zooland's roof depot in Utrecht, we have to rearrange the weekly delivery schedule to still maximize profits, as follows :

From Zooland Rotterdam :

- Deliver weekly, 4032 kgs savannah grass to Artis Amsterdam zoo, 5880 kgs savannah grass to Noorder Dierenpark Emmen zoo and 1808 kgs savannah grass to Safaripark Beekse Bergen zoo.
- Deliver weekly, 1700 kgs dutch grass only to Safaripark Beekse Bergen zoo.

From Zooland Utrecht :

- Deliver weekly, 4928 kgs dutch grass to Artis Amsterdam zoo, 8120 kgs of dutch grass to Noorder Dierenpark Emmen zoo and 6952 kgs dutch grass to Safaripark Beekse Bergen zoo.

From Extra Depot in Rotterdam :

- Deliver weekly, 5500 kgs savannah grass only to Safaripark Beekse Bergen zoo.

The profit will be fl. 2302.32, about fl. 100,- less than before, not yet counting the weekly rent of the extra depot.

VI. Conclusion

Linear programming can give a firm base in arranging profitable delivery schedules.

Reference

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Pengantar Perluasan Lapangan ... Lanjutan

V. Kesimpulan dan Saran

Telah diperlihatkan dua teknik perluasan lapangan, yaitu perluasan aljabar dan transenden. Perluasan F dari E dikatakan perluasan aljabar jika semua unsur di F adalah aljabar atas E. Jika tidak demikian dikatakan perluasan transenden.

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Pustaka

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Penulis

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