STOCHASTIC APPROACH FOR PARKING PROBLEMS ON HOOGSTRAAT ROTTERDAM*)

Agus Sukmana
Dharma Lesmono

Abstract

For the city authorities of Rotterdam, parking problem is a crucial problem that should have been solved. In order to collect data and to make a preliminary study of the real situation, the Hoogstraat is chosen as a representative of the parking bays in Rotterdam. We modeled the data obtained on December 17,1997 to solve the problems using stochastic approach. We found that M/M/29/29 loss system model is quite representative for this problem and increasing parking time by certain percentage will make the probability that all parking bays are occupied and the utilization factor become larger.

Introduction

One of the problems faced by city authorities of Rotterdam is the increasing of the number of people who did not pay the parking retribution. To deal with this problem, the city authorities of Rotterdam has arranged some policies such as

*) This paper is a part of our research project that has been executed at University of Twente during spring 1998.

allocating certain percentage of parking bays to the licensee, and introducing another way of billing system. Before applying these policies, it is necessary to make a model that describes the nature of the parking situation, and then applying these policies to the model. The model is developed based on the data taken on Wednesday December 17, 1997 at Hoogstraat [1,2].

For the car that is parked on Hoogstraat, the following items are recorded: parking bay numbers, arrival time, ticket status (buy ticket, no ticket and license), arrival time, time on ticket, departure time and the police registration of the car. License means that the owner of the car pays parking ticket monthly. Arrival and departure time are recorded by an observer. The time on the ticket states the latest point of time that a car has to depart is also recorded automatically when the driver bought ticket from the parking machine.

The Problems

The problems we want to solve are the following:

- Making a model that describes the stochastic nature of the parking situation. Based on this model we will determine the probability that all parking bays are occupied.
- The influence of assigning 25% of the parking bays to licensees to the utilization factor and determine the probability that all bays are occupied.
- Introduce another way of billing system as follows. If a car wants to leave, the driver has to pay for the time the car was parked. Unless paid, the car can not leave. We expect that the parking times of the ticket buyers (i.e. the non-parking licensees or non-licensees) will increase by certain percentage, say 10% for example. We want to answer the same problems as in the previous item.

Notations

The notations we used in this part are the following:

A/B/m	m-server queue with the inter-arrival time distribution A(t) and service time distribution B (x) identified by A and B
M	denotes exponential distribution
$p_0(t)$	P [no cars in system]
p_k	P [k cars in system]
λ	Average arrival rate
μ	Average service rate
ρ	Utilization factor

Modeling Arrival Process and Distribution of Service Time

We view the situation described above as a queueing system, with parking bays play role as servers (there are 29 parking bays). The actual parking time (differences between departure time and arrival time) is the service time in our model. We assume that the arrival time follows the Poisson process. Service time is also assumed to be independent and identically exponential distributed random variables with the same parameter. This means that the time spent by the car in the parking bays is independent with the other car because of the memoryless property of the exponential distribution. We choose M/M/29/29 (29-server loss systems), because if a customer (car) arrives when all parking bays are occupied, that customer is lost. Let p_k the probability that there are k cars in the systems, so the value of p_k is given by: [3, pp. 105]

$$p_k = \begin{cases} p_0\left(\frac{\lambda}{\mu}\right)^k \frac{1}{k!} & k \le m \\ 0 & k > m \end{cases}$$

where

$$p_0 = \left[\sum_{k=0}^{m}\left(\frac{\lambda}{\mu}\right)^k \frac{1}{k!}\right]^{-1}$$

Calculating Arrival Rate, Service Rate, Probability All Parking Bays are Occupied and Utilization factor

There are 59 data valid for observation on December 17, 1997. Among 59 data on December 17, 1997, 5 cars have the same arrival time (at 10.00). We assume that these cars have existed in the parking bays before 10.00, so these cars and the corresponding observations are excluded from the analysis.

According to the model M/M/29/29, we make several calculations in order to find the probability that all parking bays are occupied. The average arrival rate λ is calculated by counting the number of cars that arrive at the parking bays during a day and divided by the length of the observation (7 hours). We calculate the average service rate μ by taking inverse of the average actual parking time. The results are shown in the following table.

Table 1 Values Obtained by M/M/29/29 model

Values	December 17, 1997
m	29
λ	7.7143
μ	1.7841
ρ	0.1491
p_0	0.0133
P_{29}	$0.4131\ 10^{-14}$

Utilization factor here is defined as :

$$\rho = \frac{\lambda}{m\mu}$$

From Figure 1 we see that during a day the proportion of used parking bays is still always less than 1, so it is reasonable that the probability that all parking bays are occupied is very small (tends to zero).

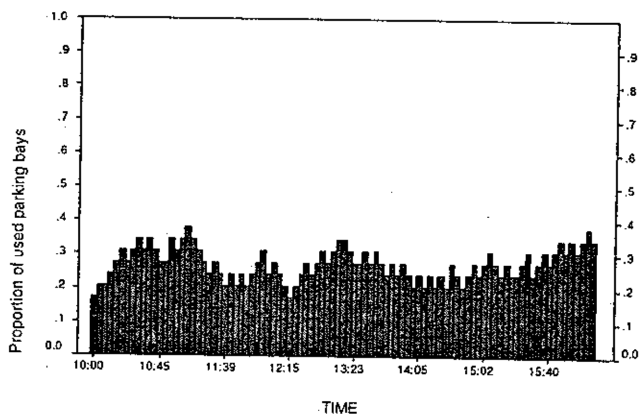

Figure 1 Proportion of Used Parking Bays

Separating Licensees and Non Licensees

In order to know the influence of assigning 25% of the parking bays (7 parking bays) to licensees, we separate the data into two groups; licensees and non-licensees. In each group we calculate the utilization factor and the probability that all bays are occupied. The results of the calculations are shown on Table 2.

Table 2 Values Obtained by Separating Licensees and Non-licensees

Values	Licensees	Non Licensees
M	7	22
λ	0.2857	7.4286
$1/\lambda$	3.5	0.1346
μ	1.4458	1.8003
$1/\mu$	0.6917	0.5554
ρ	0.0282	0.1876
p_0	0.8207	0.0161
P_m	$0.1916\ 10^{-8}$	$0.5004\ 10^{-9}$

If we compare the result on Table 1 and Table 2 we can see that by separating the licensees and non-licensees, the probability that all bays are occupied is larger than if we do not separate them. So by assigning 25% of the parking bays to the licensees, the probability that all parking bays are occupied becomes higher. From non-licensees' side, the probability that all parking bays are occupied become higher because only 22 parking bays available for them rather than before (29 parking bays). This condition also prevails for the licensees because now the licensees just using one of seven parking bays available to them. So, it is clear that for licensees, the probability that all 7 parking bays are occupied are larger than the probability that 29 parking bays are occupied.

If we assign 25% of the parking bays to licensees, the utilization factor becomes larger for the non-licensees, but lower for the licensees. For the non-licensees, this situation can be described as follows. Because the people who use the parking bays are mainly non-licensees, so by assigning 25% of parking bays to licensees the parking bays available for them become 75% than before (22 parking bays). That is why this makes the utilization factor becomes larger. For licensees, 7 parking bays are too much, because only a few people who use the parking bays, so the utilization factor becomes lower.

Other Parking Policy

We will make further calculation according to another way of billing. The new way of billing is the following: if a car wants to leave, the driver has to pay for the time the car was parked. Unless paid, the car can not leave. We expect that the parking time of the ticket buyers, i.e. the non-licensees will increase by certain percentage. We will calculate the probability that all parking bays are occupied and the influence of assigning 25% of the parking bays to the licensees for the utilization factor.

For the first scenario, we do not separate the licensee and the non-licensee, so our model still M/M/29/29. We make several calculations by increasing the parking time of the non-licensee with certain percentage from 10% up to 200%. For each percentage we calculate the probability that all parking bays are occupied and the utilization factor. We do not increase the parking time of the licensees, because licensees pay their parking time monthly. The result of this simulation is shown in Table 3.

Table 3 Values Obtained by Increasing Service Time with Certain Percentage

Values	10%	30%	50%	70%	90%	200%
m	29	29	29	29	29	29
λ	7.7143	7.7143	7.7143	7.7143	7.7143	7.7143
μ	1.6219	1.3724	1.1894	1.0495	0.9390	0.5947
ρ	0.1640	0.1938	0.2237	0.2535	0.2833	0.4473
p_0	0.0086	0.0036	0.0015	0.0006	0.0003	$0.2325\ 10^{-5}$
p_m	$0.4253\ 10^{-13}$	$0.2275\ 10^{-11}$	$0.6078\ 10^{-10}$	$0.9645\ 10^{-9}$	$0.1023\ 10^{-7}$	0.0001

From Table 3 we see that if the percentage of increasing in parking time becomes larger, the utilization factor and the probability that all parking bays are occupied become larger too. This fact is reasonable because if the parking time becomes longer and the arrival rate does not change, then the probability that the parking bays are full becomes larger. Although the probability becomes larger, but the largest value is still quite small, only 0.0001 with utilization factor 0.4473.

In the second scenario, we separate the licensees and the non-licensees and we assign 25% of the parking bays to the licensees. But, we only concern with the non-licensees because the emphasis of the billing policy is on them. This means that only 22 parking bays available for the non-licensees. We also make several calculations by increasing the parking time for the non-licensees by certain percentage and analyze the impact to the utilization factor and to the probability that all parking bays are occupied. The results are shown in the following table.

Table 4 Increasing Parking Time with respect to Licensee and Non Licensees

	Licence	Non Licence					
		10%	30%	50%	70%	90%	200%
m	7	22	22	22	22	22	22
λ	0.2857	7.4286	7.4286	7.4286	7.4286	7.4286	7.4286
μ	1.4458	1.6367	1.3849	1.2002	1.0590	0.9476	0.6001
ρ	0.0282	0.2063	0.2438	0.2813	0.3188	0.3564	0.5627
p_o	0.8207	0.0107	0.0047	0.0021	0.0009	0.0004	$0.4226\ 10^{-5}$
p_m	$0.1916\ 10^{-8}$	$0.2696\ 10^{-8}$	$0.4661\ 10^{-7}$	$0.4757\ 10^{-6}$	$0.3272\ 10^{-5}$	$0.1656\ 10^{-4}$	0.0041

Probability that all parking bays are occupied and the utilization factor for the second scenario are larger than the first scenario. The decreasing parking bays available from 29 to 22 cause this fact for the non-licensees. If we increasing the parking time by larger percentage, the probability that all parking bays are occupied and the utilization factor become larger too as what happened in the first scenario.

Another Possibility in Assigning 25% of Parking Bays to the Licensees

Instead of 7 parking bays that are reserved to the licensees, another 22 parking bays is also possible to be used by both licensee holder or non-licensees. To model this situation we draw the transition diagram on Figure 2 and Figure 3.

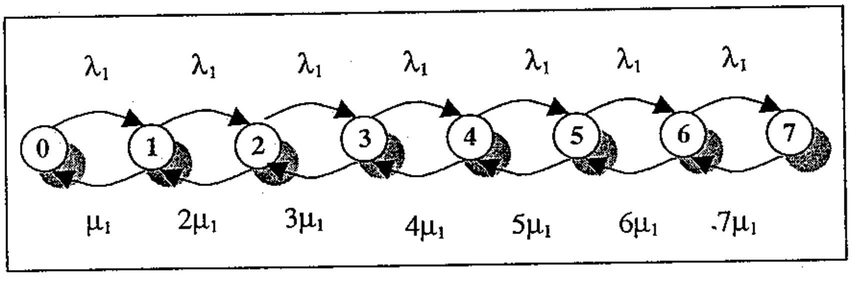

Figure 2 Transition Diagram for Reserved Parking Bays

We separate the transition diagram for 7 reserved parking bays and the other 22 parking bays. We define the variables as follows.

λ_1 : arrival rate for licensees
λ_2 : arrival rate for non-licensees
μ_1 : service rate for licensees
μ_2 : service rate for non-licensees

In our case, we are dealing with three dimensional birth-and-death queueing model. Let P(i, j, k) be the statistical equilibrium joint probability that at any instant

$$+(k+1)\mu_2 P(i,j,k+1)+\lambda_1 P(i-1,j,k)+\lambda_1\delta_1 P(i,j-1,k)+\lambda_2 P(i,j,k-$$

there are i car(s) in the 7 reserved parking bays, j cars (from licensees) and k cars (from non-licensees) in the rest of parking bays, where i=0,1,...7 ; j=0,1,..., 22 and k=0, 1,...,22 ($j + k \le 22$). We have 8 states in the reserved parking bays and 276 states in the rest of parking bays. In total our states for this model becomes 2208 states (8 times 276).

Then, equating rate out to rate in for each state (according to balance equation), the statistical equilibrium state equations are:

$$\pi_b v_b = \sum_{a \ne b} \pi_a q_{ab}$$

With $\pi_b v_b$ is the rate at which the process leaves state b and $\Sigma\, \pi_a q_{ab}$ (for $a \ne b$) is the rate at which the process enters state b. By expanding the equation above we have:

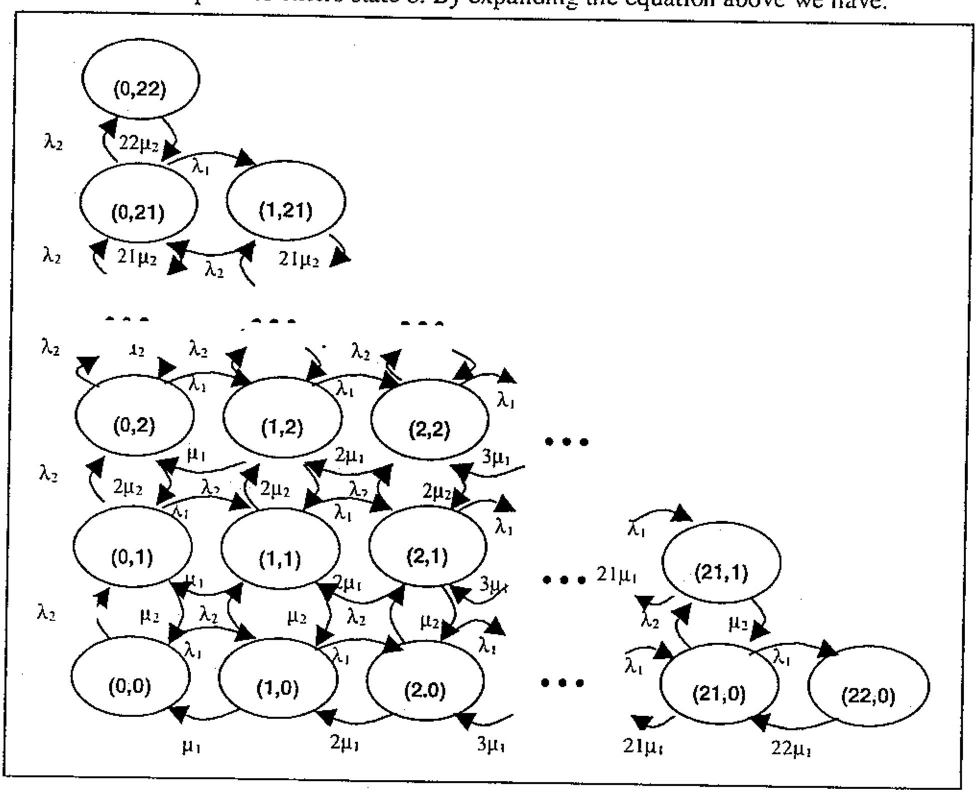

Figure 3 Transition Diagram For Non Reserved Parking Bays

$$(\lambda_1 + \lambda_2 + i\mu_1 + j\mu_1 + k\mu_2)P(i,j,k) = (i+1)\mu_1 P(i+1,j,k) + (j+1)\mu_1 P(i,j+1,k)$$
$$+ (k+1)\mu_2 P(i,j,k+1) + \lambda_1 P(i-1,j,k) + \lambda_1 \delta_1 P(i,j-1,k) + \lambda_2 P(i,j,k-1)$$

$$\sum_{\substack{0 \le i \le 7 \\ 0 \le j+k \le 22}} P(i,j,k) = 1$$

where

$$\delta_1 = I_{\{i=7\}} = \begin{cases} 1 & \text{if } i = 7 \\ 0 & \text{if } i \neq 7 \end{cases}$$

We can solve the balance equations above and find the probability that all parking bays are occupied. The utilization factor is given by the following expression.

$$\frac{1}{29}\left\{\frac{\lambda_1}{\mu_1}(1-B_1)+\frac{\lambda_2}{\mu_2}(1-B_2)\right\}$$

with B_1 is the probability that all parking bays are occupied in the reserved parking bays and B_2 is the probability that all parking bays are occupied in the non-reserved parking bays.

Conclusions

We can draw several conclusions according to the analysis of the parking data set:

- If we assign 25% of parking bays to the licensee, the probability that all parking bays are occupied and the utilization factor become larger for the non-lecensees, because the parking bays available become fewer. But for the licensee although the probability is larger, but the utilization factor is smaller.
- By assigning 25% of parking bays to the licensee, but the rest of parking bays is still probable for the licensee, our model becomes three dimensional birth-and-death queueing models. We have to solve the balance equations for 2208 states in order to obtain the probability that all parking bays are occupied. We can also find the value of utilization factor according to the given expression.

- If we increase the parking time by certain percentage, the probability that all parking bays are occupied and the utilization factor become larger.
- Our model M/M/29/29 loss system is quite representative for the parking problem.

References

Sukmana, A. and Lesmono, D. 1998. *Using Statistical Techniques for Exploring Parking Data on Hoogstraat Rotterdam*. Integral. Vol. 3.

Sukmana, A. and Lesmono, D. June 1999. *Mathematical Modeling for Parking Problems*. Internal Report FMIPA Unpar.

Kleinrock, L. 1975. *Queueing Systems Volume I: Theory*, John Wiley & Sons.

Curriculum Vitae

Drs. Agus Sukmana, MSc and **Drs. Julius Dharma Lesmono, SE, MT, MSc** are lecturers at Mathematics Department Parahyangan Catholic University.