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# Mathematical Modelling for Human Resources Allocation Problem


*Abstract*

*Human Resources allocation problem is concerned with arranging for the right number of persons to be allocated to various tasks. This article deals with an example how to assign 7 person to 7 tasks by considering 4 level of preference and time availability. The optimal solution is solved by using LINDO and LINGO.*



*Keywords:*

*zero-one integer programming, allocation problem, LINDO-LINGO.*



*Intisari*

*Masalah pengalokasian sumber daya manusia berhubungan dengan penugasan sejumlah orang yang tepat pada tugas yang tersedia. Artikel ini membahas sebuah contoh bagaimana menugaskan 7 orang pada 7 tugas dengan mempertimbangkan preferensi masing-masing dan ketersediaan waktu. Solusi optimal diselesaikan dengan bantuan perangkat lunak LINDO dan LINGO.*



*Kata-kunci:*

*pemrograman bilangan buat biner, masalah pengalokasian, LINDO-LINGO.*


## I. Introduction

The human resources allocation problem is concerned with arranging for the right number of persons to be allocated to various tasks. It is about what a person should do in a certain time period, not how to carry out task. In general we need to consider the following aspects: qualification, preference, and availability. Qualification means that a person has ability and capability to do the task. Which task does a person prefers to do and when a person is available to do the task are considered as preference and availability.

This article deals with how to assign people to various tasks by considering preference and availability aspects.

## II. Problem and Assumptions

There are 7 persons, 7 tasks, and 5 days. Each person has to work whole weeks (Monday till Friday) unless not available. Each task takes half a day (one unit of time) and one person performs one task at a time, but task 2 and task 5 need 2 persons. A and B only work four days, D is unavailable or free on Wednesday and G is unavailable or free on Friday. Degree of preference, units of time per week available, and how often a task should be performed each day are provided in the following table.

| Task / Person | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Units of time/week available |
|---|---|---|---|---|---|---|---|---|
| A | ++ | 0 | - | + | 0 | - | - | 8 |
| B | 0 | ++ | + | + | 0 | + | - | 8 |
| C | + | - | ++ | ++ | 0 | - | 0 | 10 |
| D | 0 | + | - | 0 | ++ | + | - | 8 |
| E | + | + | - | 0 | ++ | ++ | + | 10 |
| F | + | - | + | - | + | + | ++ | 10 |
| G | ++ | + | ++ | - | + | 0 | + | 8 |
| Each day | 3 | 2 | 4 | 1 | 2 | 4 | $\infty$ | |

++ Most preferred + preferred 0 indifferent - less prefer

Task 7 will be performed if there are still units of time available in a day. We will give a schedule starting on Monday that contains which tasks should be performed in a week and who will perform those tasks. We assume that if we decide to perform a certain task on a certain day, that task must be completely performed. For example, if we decided to perform task 1 at certain day, then task 1 must be performed three times that day. Tasks that can not be performed in a day must be performed on the next day. For instance, if task 1 is not performed on Monday, task 1 must be performed on Tuesday.

In this article we will solve the following problems based on our assumptions.

- Determining which tasks should be performed on each day.
- Assigning people to those tasks in accordance with his preference. We also determine on which day A and B should be free in order to obtain the optimal solution.

## III. Determine Which Tasks Should be Performed

From the table above, we know that not all tasks can be performed. There are only 62 units of time available in a week, but all tasks need at least 100 units of time (if task 7 is never performed during a week). We have to determine which tasks should be performed on each day during a week. We will solve this problem by using *zero-one integer programs*. Our objective function is to maximize the number of finished task (including its frequencies) on each day. At first we start on Monday with three possibilities: A and B are free, A or B is free and all the other people work. Each of these possibilities has consequences in the number of units of time available on Monday. Then based on each possibility we continue to Tuesday till Friday. It is obvious that given those three possibilities on Monday there are more possibilities on Tuesday, Wednesday, Thursday, and Friday. Finally there are 15 possible alternatives from Monday to Friday. From each possibility on each day we determine which task should be done. For example if one takes the possibility that A and B are free on Monday then there are 10 units of time available, because there are only 5 persons working. We define the variables for i=1, 2,...,6 as follows.

$x_i = 1$ if task i is performed on Monday
$x_i = 0$ if task i is not performed on Monday
Our zero-one integer program becomes:

$$\text{Max } 3x_1+4x_2+4x_3+x_4+4x_5+4x_6$$
$$\text{s.t. } 3x_1+4x_2+4x_3+x_4+4x_5+4x_6 \leq 10$$
$$x_i = 0 \text{ or } 1 \quad i=1,2,...,6$$

By using LINDO, we get $x_1 = x_2 = x_3 = 0$ and $x_4 = x_5 = x_6 = 1$. This means if A and B are free on Monday, we perform task 4, 5 and 6. These tasks require 9 units of time, so there is still one unit of time available and this is used to perform task 7. In LINDO, the formulation and the optimal solution are as follows.

```
MAX 3X1+4X2+4X3+X4+4X5+4X6
SUBJECT TO
   2) 3X1+4X2+4X3+X4+4X5+4X6<=10
END
```

**Table 1. All Possible Alternatives in Determining Which Task Should be Performed in Each Day**

| Alternative | Monday | Tuesday | Wednesday | Thursday | Friday | Total | [illegible] | [illegible] |
|---|---|---|---|---|---|---|---|---|
| 1 | A&B free (10) | All work (14) | D is free (12) | All work (14) | G is free (12) | 62 | 2 | 48 |
| | $X_1=X_2=X_3=0$ (9) | $X_5=X_6=0$ (12) | $X_2=X_3=0$ (12) | $X_1=X_5=0$ (13) | $X_2=X_3=0$ (12) | 58 | | |
| 2 | A/B free (12) | A/B free (12) | D is free (12) | All work (14) | G is free (12) | 62 | [illegible] | 49 |
| | [illegible] (12) | [illegible] (12) | [illegible] (12) | $X_1=X_5=0$ (13) | [illegible] (12) | 61 | | |
| 3 | A/B free (12) | All work (14) | D& (A/B) free (10) | All work (14) | G is free (12) | 62 | 4 | 48 |
| | $X_2=X_3=0$ (12) | $X_1=X_5=0$ (13) | $X_2=X_3=X_6=0$ (8) | $X_1=X_5=0$ (13) | $X_2=X_3=0$ (12) | 58 | | |
| 4 | A/B free (12) | All work (14) | D is free (12) | A/B free (12) | G is free (12) | 62 | [illegible] | 49 |
| | [illegible] (12) | [illegible] (13) | [illegible] (12) | [illegible] (12) | [illegible] (12) | 61 | | |
| 5 | A/B free (12) | All work (14) | D is free (12) | All work (14) | G&(A/B) free (10) | 62 | 4 | 48 |
| | $X_2=X_3=0$ (12) | $X_1=X_5=0$ (13) | $X_2=X_3=0$ (12) | $X_1=X_5=0$ (13) | $X_2=X_3=X_6=0$ (8) | 58 | | |
| 6 | All work (14) | A&B free (12) | D is free (12) | All work (14) | G is free (12) | 64 | 5 | 47 |
| | $X_1=X_2=0$ (13) | $X_3=X_5=X_6=0$ (8) | $X_1=X_2=X_4=0$ (12) | $X_3=X_5=0$ (12) | $X_1=X_2=X_4=0$ (12) | 57 | | |
| 7 | All work (14) | A/B free (12) | D& (A/B) free (10) | All work (14) | G is free (12) | 62 | 4 | 48 |
| | $X_1=X_2=0$ (13) | $X_3=X_5=0$ (12) | $X_1=X_2=X_6=0$ (9) | $X_3=X_5=0$ (12) | $X_1=X_2=X_4=0$ (12) | 58 | | |
| 8 | All work (14) | A/B free (12) | D is free (12) | A/B free (12) | G is free (12) | 62 | [illegible] | [illegible] |
| | [illegible] (13) | [illegible] (12) | [illegible] (12) | [illegible] (12) | [illegible] (12) | [illegible] | | |
| 9 | All work (14) | A/B free (12) | D is free (12) | All work (14) | G is free (12) | 64 | 4 | 48 |
| | $X_1=X_2=0$ (13) | $X_3=X_5=0$ (12) | $X_1=X_2=X_4=0$ (12) | $X_3=X_5=0$ (12) | $X_1=X_2=X_6=0$ (9) | 58 | | |
| 10 | All work (14) | All work (14) | D,A,B free (8) | All work (14) | G is free (12) | 62 | 5 | 47 |
| | $X_1=X_2=0$ (13) | $X_5=X_6=0$ (12) | $X_1=X_2=X_3=X_4=0$ (8) | $X_5=X_6=0$ (12) | $X_2=X_3=0$ (12) | 57 | | |
| 11 | All work (14) | All work (14) | D& (A/B) free (10) | A/B free (12) | G is free (12) | 62 | 4 | 48 |
| | $X_1=X_2=0$ (13) | $X_5=X_6=0$ (12) | $X_1=X_2=X_3=0$ (9) | $X_5=X_6=0$ (12) | $X_2=X_3=0$ (12) | 58 | | |
| 12 | All work (14) | All work (14) | D& (A/B) free (10) | All work (14) | G&(A/B) free (10) | 62 | 7 | 45 |
| | $X_1=X_2=0$ (13) | $X_5=X_6=0$ (12) | $X_1=X_2=X_3=0$ (9) | $X_5=X_6=0$ (12) | $X_1=X_2=X_3=0$ (9) | 55 | | |
| 13 | All work (14) | All work (14) | D is free (12) | A&B free (10) | G is free (12) | 62 | 4 | 48 |
| | $X_1=X_2=0$ (13) | $X_5=X_6=0$ (12) | $X_2=X_3=0$ (12) | $X_1=X_5=X_6=0$ (9) | $X_2=X_3=0$ (12) | 58 | | |
| 14 | All work (14) | All work (14) | D is free (12) | A/B free (12) | G&A/B free (10) | 62 | 5 | 47 |
| | $X_1=X_2=0$ (13) | $X_5=X_6=0$ (12) | $X_2=X_3=0$ (12) | $X_1=X_4=X_5=0$ (12) | $X_2=X_3=X_6=0$ (8) | 57 | | |
| 15 | All work (14) | All work (14) | D is free (12) | All work (14) | G,A,B free (8) | 62 | 4 | 48 |
| | $X_1=X_2=0$ (13) | $X_5=X_6=0$ (12) | $X_2=X_3=0$ (12) | $X_1=X_5=0$ (13) | $X_2=X_3=X_6=0$ (8) | 58 | | |

```
INTE 6

OBJECTIVE FUNCTION VALUE

        1)        9.000000

  VARIABLE        VALUE      REDUCED COST
        X1     0.000000        -3.000000
        X2     0.000000        -4.000000
        X3     0.000000        -4.000000
        X4     1.000000        -1.000000
        X5     1.000000        -4.000000
        X6     1.000000        -4.000000

  ROW   SLACK OR SURPLUS    DUAL PRICES
   2)          1.000000       0.000000

 NO. ITERATIONS=        7
 BRANCHES=     0 DETERM.=  1.000E     0

 NO. ITERATIONS=        7
 BRANCHES=     0 DETERM.=  1.000E     0
```

INTE 6 in the formulation above means that the first six variables are 0-1 variables. In solving the zero-one integer programs for Tuesday, given that A and B are free on Monday, we use the same formulation but we set $x_1 = x_2 = x_3 = 1$ because task 1, 2 and 3 are not performed on Monday. We use LINDO to solve the similar integer programming for every possibility we have. The result of all these alternatives from Monday till Friday are provided in table 1.

In order to determine which alternative we choose, we calculate the unit(s) of time differences in a week between units of time available (according to each alternative) and units of time used to perform task 1 up to task 6 (as a result of our zero-one integer programs using LINDO). Actually the differences represent units of time that are used to perform task 7, because task 7 is performed if there are still unit(s) of time available.

From table 1 we see that alternative 1, 2, and 8 give the minimum differences, but we still have to choose between alternative 1, 2, and 8 for the best alternative. To make a choice between those three alternatives, we look at the maximum number of finished tasks in a week. The numbers of finished tasks are calculated by summing up all the objective values for each alternative (second row of each alternative). From the last column of table 1, the maximum number of finished task is 51 and this is given by alternative 8, so we choose it as the best alternative. Table 2 gives list of tasks that should be performed according to alternative 8.

**Table 2. List of Tasks Should be Done According to Scenario 8**

| Task / Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Note |
|---|---|---|---|---|---|---|---|---|
| Monday | - | - | √ | √ | √ | √ | √ | **All work** |
| Tuesday | √ | √ | - | √ | - | √ | - | **A/B is free** |
| Wednesday | - | - | √ | - | √ | √ | - | **D is free** |
| Thursday | √ | √ | √ | √ | - | - | - | **A/B is free** |
| Friday | √ | - | - | √ | √ | √ | - | **G is free** |

## IV. Assigning People to Tasks

After knowing which tasks should be done on each day, we start to assign people to those tasks for each day. First we convert the preference of people to certain tasks into a number. We convert ++ to 1, + to 2, 0 to 3 and – to 4. Then for each day we formulate a zero-one integer program to minimize the preference value. In general the formulation is the same as the formulation for the previous assignment problem. In our case, we arrange the problem for each day such that the assignment problem becomes balanced, i.e. the number of persons and the number of tasks are the same. To do this, we split each task into its frequencies (per day) and split the person into two units of time (for instance, A is split into A1 and A2). For example we want to assign people to tasks on Monday as shown in Table 3.

On Monday, all people work to do task 3, 4, 5, 6, and 7. It means we have 14 unit of time and it is represented by A1 up to G2. Each task on Monday is also split in its frequencies, for instance task 3 must be performed four times, so we have 3.1, 3.2, 3.3 and 3.4. There are represented by task numbers 1 to 14 as well.

**Table 3. Assignment on Monday (all work)**

| Task | | 3 | | | | 4 | 5 | | | | 6 | | | | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 3.1 | 3.2 | 3.3 | 3.4 | 4.1 | 5.1 | 5.2 | 5.3 | 5.4 | 6.1 | 6.2 | 6.3 | 6.4 | 7.1 |
| A | A1 | 4 | 4 | 4 | 4 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 |
| | A2 | 4 | 4 | 4 | 4 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 |
| B | B1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 4 |
| | B2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 4 |
| C | C1 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 3 |
| | C2 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 3 |
| D | D1 | 4 | 4 | 4 | 4 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 |
| | D2 | 4 | 4 | 4 | 4 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 |
| E | E1 | 4 | 4 | 4 | 4 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| | E2 | 4 | 4 | 4 | 4 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| F | F1 | 2 | 2 | 2 | 2 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| | F2 | 2 | 2 | 2 | 2 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| G | G1 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 |
| | G2 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 |

Thus we have for Monday,

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | A1 | A2 | B1 | B2 | C1 | C2 | D1 | D2 | E1 | E2 | F1 | F2 | G1 | G2 |
| Task | 3.1 | 3.2 | 3.3 | 3.4 | 4.1 | 5.1 | 5.2 | 5.3 | 5.4 | 6.1 | 6.2 | 6.3 | 6.4 | 7.1 |

We have 14 units of time and 14 frequencies of tasks to be matched in order to minimize the preference value. We must determine which person (unit of time) should be assigned to each task. We define the variables as follows.

$x_{ij}$ = 1 if unit of time i is assigned to a frequency of task j

$x_{ij}$ = 0 if unit of time i is not assigned to a frequency of task j

$c_{ij}$ = the preference value of unit time i to a frequency of task j

$c_{ij} \in \{1,2,3,4\}$

On Monday, the formulation of the integer programming becomes:

$$Min\ z = \sum_{j=1}^{14}\sum_{i=1}^{14} c_{ij}x_{ij}$$

$$\sum_{j=1}^{14} x_{ij} = 1 \quad j = 1, 2, \ldots 14$$

$$\sum_{i=1}^{14} x_{ij} = 1 \quad i = 1, 2, \ldots 14$$

$$x_{ij} = 0 \text{ or } 1$$

The first constraint ensures that each unit of time is assigned to a task and the second ensures that each task is completed. Using LINGO, the formulation becomes:

```
MODEL:
SETS:
PERSONS/1..14/;
TASKS/1..14/;
LINKS (PERSONS, TASKS):PREF, ASSIGN;
ENDSETS
MIN=@SUM(LINKS: PREF*ASSIGN);
@FOR (PERSONS(I):
@SUM (TASKS(J):ASSIGN(I,J))<1);
@FOR (TASKS(J):
@SUM PERSONS(I):ASSIGN(I,J))>1);
DATA:
PREF = 4,4,4,4,2,3,3,3,3,4,4,4,4,4
4,4,4,4,2,3,3,3,3,4,4,4,4,4,
2,2,2,2,2,3,3,3,3,4,4,4,4,4,
2,2,2,2,2,3,3,3,3,4,4,4,4,4,
1,1,1,1,1,3,3,3,3,4,4,4,4,3,
1,1,1,1,1,3,3,3,3,4,4,4,4,3,
4,4,4,4,3,1,1,1,1,2,2,2,2,4,
4,4,4,4,3,1,1,1,1,2,2,2,2,4,
4,4,4,4,3,1,1,1,1,1,1,1,1,2,
4,4,4,4,3,1,1,1,1,1,1,1,1,2,
2,2,2,2,4,2,2,2,2,2,2,2,2,1,
2,2,2,2,4,2,2,2,2,2,2,2,2,1,
```

```
24] 1,1,1,1,4,2,2,2,2,3,3,3,3,2,
25] 1,1,1,1,4,2,2,2,2,3,3,3,3,2;
26] ENDDATA
27] END
```

Line 2 defines the 14 units of time and line 3 describes the 14 frequencies of tasks. In line 4 we define each possible combination of unit of time and frequencies of tasks and associate them with each combination of an assignment preference value and a variable ASSIGN (I,J). ASSIGN (I,J) equals 1 if unit of time I is used to perform task J and equals 0 otherwise. Line 5 ends the definition of sets. Line 6 expresses the objective function by summing over all possible (I,J) combinations, the product of the preference value and ASSIGN(I,J). Lines 7-8 limit each PERSONS to perform at most one task by forcing (for each unit of time) the sum of ASSIGN(I,J) over all TASKS to be at most 1. Lines 9-10 require that each TASKS be completed by forcing (for each task) the sum of ASSIGN(I,J) over all PERSONS to be at least 1. Lines 12-26 consists of the input of the preference values.

The non-zero solutions are given below as follows:

| Variable | Value | Meaning |
|---|---|---|
| ASSIGN ( 1, 6) | 1.000000 | A (1) is assigned to task 5 (6) |
| ASSIGN ( 2, 8) | 1.000000 | A (2) is assigned to task 5 (8) |
| ASSIGN ( 3, 5) | 1.000000 | B (1) is assigned to task 4 (5) |
| ASSIGN ( 4, 4) | 1.000000 | B (2) is assigned to task 3 (4) |
| ASSIGN ( 5, 2) | 1.000000 | C (1) is assigned to task 3 (2) |
| ASSIGN ( 6, 3) | 1.000000 | C (2) is assigned to task 3 (3) |
| ASSIGN ( 7, 9) | 1.000000 | D (1) is assigned to task 5 (9) |
| ASSIGN ( 8, 7) | 1.000000 | D (2) is assigned to task 5 (7) |
| ASSIGN (9,12) | 1.000000 | E (1) is assigned to task 6 (12) |
| ASSIGN (10,10) | 1.000000 | E (2) is assigned to task 6 (10) |
| ASSIGN (11,13) | 1.000000 | F (1) is assigned to task 6 (13) |
| ASSIGN (12,11) | 1.000000 | F (2) is assigned to task 6 (11) |
| ASSIGN (13, 1) | 1.000000 | G (1) is assigned to task 3 (1) |
| ASSIGN (14,14) | 1.000000 | G (2) is assigned to task 7 (14) |

This means that on Monday, A is assigned to task 5, B to task 5 and 4, C to task 3, D to task 5, E and F to task 6 and G to task 3 and 7 with mininum preference value is 49. We do the same procedure for the other day, and the result are provided in tables 4, 5, 6, 7, 8, and 9 where the shaded areas in each table (in the preference value) are the optimal assignments for that day. We make two calculations on Tuesday and Thursday in order to decide whether A or B is free on that day. In table 10, we see that if A is free on Tuesday the total preference value of alternative 8 is 95 but if A is free on Thursday the value is 97. Because we want to minimize the preference value, we let A to be free on Tuesday. So the optimal solution for this problem is to follow alternative 8 with A free on Tuesday as shown in table 11. Numbers in bracket in table 11 indicate the frequencies of tasks that should be done that day.

**Table 4. Assignment on Tuesday 1 (A is free)**

| Task | | 1 | | | 2 | | | | 4 | 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 1.1 | 1.2 | 1.3 | 2.1 | 2.2 | 2.3 | 2.4 | 4.1 | 6.1 | 6.2 | 6.3 | 6.4 |
| B | B1 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |
| | B2 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |
| C | C1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 4 | 4 | 4 | 4 |
| | C2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 4 | 4 | 4 | 4 |
| D | D1 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2 |
| | D2 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2 |

Table 4. Assignment on Tuesday 1 (A is free) [continued]

| Task | | 1 | | | 2 | | | | 4 | 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 1.1 | 1.2 | 1.3 | 2.1 | 2.2 | 2.3 | 2.4 | 4.1 | 6.1 | 6.2 | 6.3 | 6.4 |
| E | E1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | [illegible] | 1 | 1 | 1 |
| | E2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 |
| F | F1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 |
| | F2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 |
| G | G1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 | 3 | 3 | 3 | 3 |
| | G2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 | 3 | 3 | 3 | 3 |

Table 5. Assignment on Tuesday 2 (B is free)

| Task | | 1 | | | 2 | | | | 4 | 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 1.1 | 1.2 | 1.3 | 2.1 | 2.2 | 2.3 | 2.4 | 4.1 | 6.1 | 6.2 | 6.3 | 6.4 |
| A | A1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 2 | 4 | 4 | 4 | 4 |
| | A2 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 2 | 4 | 4 | 4 | 4 |
| C | C1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 4 | 4 | 4 | 4 |
| | C2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 4 | 4 | 4 | 4 |
| D | D1 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2 |
| | D2 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2 |
| E | E1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 |
| | E2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 |
| F | F1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 |
| | F2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 |
| G | G1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 | 3 | 3 | 3 | 3 |
| | G2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 | 3 | 3 | 3 | 3 |

Table 6. Assignment on Wednesday (D is free)

| Task | | 3 | | | | 5 | | | | 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 3.1 | 3.2 | 3.3 | 3.4 | 5.1 | 5.2 | 5.3 | 5.4 | 6.1 | 6.2 | 6.3 | 6.4 |
| A | A1 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| | A2 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| B | B1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| | B2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| C | C1 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| | C2 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| E | E1 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | E2 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| F | F1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | F2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| G | G1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| | G2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |

Table 7. Assignment on Thursday 1 (B is free)

| Task | | 1 | | | 2 | | | | 3 | | | | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 1.1 | 1.2 | 1.3 | 2.1 | 2.2 | 2.3 | 2.4 | 3.1 | 3.2 | 3.3 | 3.4 | 4.1 |
| A | A1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 2 |
| | A2 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 2 |
| C | C1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| | C2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| D | D1 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| | D2 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| E | E1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| | E2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| F | F1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 | 4 |
| | F2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 | 4 |
| G | G1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 4 |
| | G2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 4 |

Table 8. Assignment on Thursday 2 (A is free)

| Task | | 1 | | | 2 | | | | 3 | | | | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 1.1 | 1.2 | 1.3 | 2.1 | 2.2 | 2.3 | 2.4 | 3.1 | 3.2 | 3.3 | 3.4 | 4.1 |
| B | B1 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |
| | B2 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |
| C | C1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| | C2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| D | D1 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| | D2 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| E | E1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| | E2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 3 |
| F | F1 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 | 4 |
| | F2 | 2 | 2 | 2 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 | 4 |
| G | G1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 4 |
| | G2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 4 |

Table 9. Assignment on Friday (G is free)

| Task | | 1 | | | 4 | 5 | | | | 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Person | | 1.1 | 1.2 | 1.3 | 4.1 | 5.1 | 5.2 | 5.3 | 5.4 | 6.1 | 6.2 | 6.3 | 6.3 |
| A | A1 | 1 | 1 | 1 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| | A2 | 1 | 1 | 1 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| B | B1 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| | B2 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| C | C1 | 2 | 2 | 2 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| | C2 | 2 | 2 | 2 | 1 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| D | D1 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| | D2 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| E | E1 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | E2 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| F | F1 | 2 | 2 | 2 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | F2 | 2 | 2 | 2 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

**Table 10. Comparing Alternatives With Respect to Scenario 8**

| | Minimax Preference Value | | | | | |
|---|---|---|---|---|---|---|
| Scenario 8 | Monday | Tuesday | Wednesday | Thursday | Friday | Total |
| A is free on Tuesday | 23 | 17 | 20 | 18 | 17 | [illegible] |
| A is free on Thursday | 23 | 19 | 20 | 18 | 17 | 97 |

**Table 11. Optimal Assignment in a Week**

| Task / Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Monday | - | - | B(1),C(2),G(1) | B(1) | A(2),D(2) | E(2),F(2) | G(1) |
| Tuesday | C(1),G(2) | B(2),D(2) | - | C(1) | - | E(2),F(2) | - |
| Wednesday | - | - | C(2),G(2) | - | A(2),E(2) | B(2),F(2) | - |
| Thursday | A(2),G(2) | D(2),E(2) | C(1),F(2),G(1) | C(1) | - | - | - |
| Friday | A(2),C(1) | - | - | C(1) | D(2),E(1),F(1) | B(2),E(1),F(1) | - |

## V. Conclusion

We can draw several conclusions based on the problem:

- The optimal assignment is obtained when A and B are not free on the same day and each person tends to do the same task in a day in accordance with his preference.
- According to the optimal solution, there are at least 6 people who work on a day.
- For further analysis, it is interesting if we can include qualification aspects in this problem, but of course we need more information for this purpose. Another interesting aspect concerns the number of tasks that can not be performed everyday because available time constraints. If more data are available, we can e.g. consider the way to fulfill those tasks by either using the people who have to work overtime or hiring other people.

## VI. References


[1] Salkin, H.M and Mathur K, "*Foundations of Integer Programming*", North-Holland, 1989.

[2] Winston, W.L, "*Operation Research: Applications and Algorithms*", third edition, Duxbury Press, California, 1994.



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