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**A STUDY ON  
TASK SPECIFICATION AND CONTROL  
FOR  
TWO COOPERATING ROBOT ARMS**

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# Abstract

This thesis focuses on *task specification and control for two cooperating robot arms*. A new task specification is designed to allow a multi-robot system, in this case two cooperating industrial robots, to perform a cooperative task in a coordinated manner. The coordinated motion between the two robots is realized by feedforwarding the velocity trajectories of the first robot to the second robot, or vice versa. Unfortunately, this technique leads to a non-practical solution of having to specify tasks for both robots.

Therefore, another task specification is introduced by exploiting the redundant properties of the multi-robot system. This specification is an extension of an existing task specification for a single robot arm. Redundancy is used mostly for executing tasks which consist of a relative motion between both robots. This method is straightforward, since the programmer only has to focus on the contact and relative motion between the tool, held by one robot, and the manipulated object, held by the other robot.

The utilization of redundant properties of the multi-robot system is studied. Redundant manipulator theory is applied to both cooperating robots so that during the task execution both robots stay within a reachable and comfortable workspace, keeping them away from their singular positions. Such an approach allows the multi-robot system to perform a complete operation in one step; even difficult tasks can be accomplished easily for which most single robot arms fail.

The above theory is demonstrated using simulation studies which show very interesting results by utilizing redundancy of the multi-robot system. Finally some experiments were performed using two industrial robots, one of which was equipped with a six-dimensional force sen-

sor. A comparison of some experimental results using a single robot arm and two cooperating robot arms has been made and an effective method for specifying the cooperative task based on redundancy is proven.



# Symbols and definitions

- $\equiv$  : the left-hand side is a shorthand notation for the expression on the right-hand side
- $\Delta$  : difference between the desired and measured value
- $\overset{str}{\Leftarrow}$  : transformation from the right-hand side expression to the left-hand side expression
- $\bullet_d$  : desired value of variable or vector
- $\bullet_m$  : measured value of variable or vector
- $\delta$  : the half of rotation angle in determining relative position between two robots
- $\psi$  : rotation angle in determining direction of the KUKA-160 IR w.r.t the KUKA-361 IR
- $\tau$  : a half rotation of an equivalent fifth axis (used in spherical trigonometric computation)
- $\theta$  : generalized joint position
- $\theta, \dot{\theta}, \ddot{\theta}$  : column vector of generalized joint positions and its derivatives with respect to time:

$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}, \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}, \ddot{\theta} = \begin{bmatrix} \ddot{\theta}_1 \\ \vdots \\ \ddot{\theta}_n \end{bmatrix}$$

$\theta_{min}, \theta_{max}$  : lower and upper joint limits

$\omega$  : angular velocity three-vector:

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$a_i, \alpha_i, d_i, \theta_i$  : link's parameters according to the **Denavit-Hartenberg** convention

$a$  : scalar (unbold lowercase)

$\mathbf{a}$  : column vector (bold lowercase)

$act$  : actual value (subscript)

$axt, ayt, azt$  : task frame orientations along coordinate axes of a reference frame (used in Comrade command file)

$b_i$  : base frame of the  $i$ -th robot

$e_i$  : end-effector frame of the  $i$ -th robot

$ef$  : end-effector frame (superscript, subscript)

$f$  : force portion (subscript)

$f$  : forward kinematics

$f^{-1}$  : inverse kinematics

$ff$  : feedforward (subscript)

$f$  : force:

$$f = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$j1 \dots j6$  : joint positions (used in Comrade command file)

$k, k_*$  : constant (unbold lowercase)

$m$  : moment (torque):

$$m = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

$n$  : constant (unbold lowercase)

$o$  : environment frame (subscript)

$out$  : output (subscript)

$p$ : position portion (subscript)  
 $\mathbf{p}$ : position vector:

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$\mathbf{p} \times \mathbf{a}$ : cross product of three-vectors  
 $[\mathbf{p} \times]$ : matrix representing cross product with vector  $\mathbf{p}$ :

$$[\mathbf{p} \times] = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}$$

$\mathbf{r}$ : orientation vector:

$$\mathbf{r} = \begin{bmatrix} r_\rho \\ r_\theta \\ r_\psi \end{bmatrix}$$

$s$ : sensor frame (superscript, subscript)  
 $s$ : Laplace transform variable (unbold lowercase)  
 $t$ : time (unbold lowercase)  
 $t, tf$ : task frame (superscript, subscript)  
 $tot$ : total (subscript)  
 $\mathbf{u}$ : control commands, generated by the control law  
 $v$ : velocity portion (subscript)  
 $\mathbf{v}$ : translational (or rectilinear) velocity three-vector:

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$\mathbf{w}, \mathbf{y}$ : arbitrary vector

$x$  : manipulation variable:

$$x = \begin{bmatrix} p \\ r \end{bmatrix}$$

$x_d, \dot{x}_d, \ddot{x}_d$  : desired end-effector position coordinates, and its derivative with respect to time

$x_t, y_t, z_t$  : task frame positions along coordinate axes of a reference frame (used in Comrade command file)

$x_r, y_r$  : distance between two robot base frames in XY-plane

$A$  : matrix (bold uppercase)

$A^T$  : transpose of matrix  $A$

$A^\dagger$  : pseudo-inverse of matrix  $A$

$F$  : generalized force vector :

$$F = \begin{bmatrix} f \\ m \end{bmatrix}$$

$I$  : identity matrix

$J$  : the Jacobian (matrix mapping joint velocities to cartesian velocities):

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J\dot{\theta}$$

$J^{-1}$  : inverse of the Jacobian

$K_o$  : stiffness matrix of the compliant structure

$K_f$  : force control gain

$K_p$  : position control gain

$K_{tr}$  : tracking control gain

$N(J)$  : null space of the Jacobian matrix

$P_f$  : potential function

$P_j$  : potential function in joint space

$P_c$  : potential function in cartesian space

- $P_{shl}$  : potential function for avoiding singularity above robot shoulder  
 $R(J)$  : range space of the Jacobian matrix  
 $\mathcal{R}(J)$  : set of real numbers  
 $S_M$  : manipulable space  
 $S_R$  : redundant space  
 $S$  : selection matrix  
 ${}^o_nS$  :  $6 \times 6$  screw transformation matrix (transformation of generalized velocity vector) from frame  $\{o\}$  to frame  $\{n\}$ :

$${}^nV = {}^o_nS {}^oV,$$

$${}^o_nS = \begin{bmatrix} {}^o_nR & [p^{n,o} \times] {}^o_nR \\ O_{3 \times 3} & {}^o_nR \end{bmatrix}$$

- ${}^o_nT$  :  $4 \times 4$  homogeneous transformation matrix; maps coordinates of three-vectors from reference frame  $\{o\}$  to reference frame  $\{n\}$ :

$${}^o_nT = \begin{bmatrix} {}^o_nR & {}^o_np \\ O_{1 \times 3} & 1 \end{bmatrix},$$

${}^o_np$  is the position vector of  $\{o\}$ 's origin in reference frame  $\{n\}$ .

- $V$  : generalized velocity vector:

$$V = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- $X, Y, Z$  : coordinate axes of right-handed orthogonal reference frame



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# Chapter 1

## Introduction

Robotics technology is becoming increasingly important in industry and incorporated into manufacturing systems. This is due to the increased number of potential applications of robots on manufacturing floors as well as in other areas. Robotic systems hold a great promise for assisting in many endeavors. For example, robotised automation of complicated tasks is becoming a very important aspect in manufacturing and in assembly operations. This tendency is recognized by looking at the rapid development and extensive research on control techniques which aim at utilizing robot manipulators in a more flexible and optimal way.

Robot manipulators may be able to work perfectly in environments which are not suitable for human beings. Hazardous, tedious, and repetitive jobs can be accomplished safely and accurately using robot manipulators. Robots are also capable of performing or (re)producing exactly the same motions, and they can cope with inaccurate information about the environment. These advantages increase the manufacturing flexibility, but at the same time they require more powerful systems with greater adaptability to the variation of process conditions.

First-generation robots were only provided with position control, as they were utilized mostly for '*pick and place*' operations. In practice however, these operations are not representative for most applications, and manipulators are very often constrained by their environment. In such cases, contact between robot or manipulated object and environment has to be dealt with instead of to be avoided. Consequently,

problems arise that are related to control stability, robustness, accuracy and so on.

Following very long research activities in this field, the robotics research community has realized that more intelligent robot manipulators must be developed. This thought triggered the researchers to improve and increase the autonomy of the robots. This can be realized for instance by means of applying a variety of sensors, such as force, vision, tactile, ultra-sound, infra-red sensors etc. Hence, the robot interacts with its environment in a more intimate fashion, similar as a human being who can use all kinds of sensors to capture and utilize incoming information as much as possible. The more sensors that are applied, the more autonomous is the robot.

The evolution of robotic operations began with very simple operations, e.g. point-to-point handling operations, but later moved towards complex operations where a variety of incoming stimuli are utilized through sensor measurements. Such a complex operation can be found in remote assembly operations using a multi-robot system [72]. All incoming information should be processed in order to enable the robots to work in a more intelligent way. Due to the use of sensors and advanced controllers, the robot becomes more intelligent and autonomous. It can cope with the errors of the positioning system, with the incompleteness of the information about the environment, e.g. the uncertainty of the geometric model of the environment, etc.

A typical example of a simple robot operation is transfer of an object, from one position to an other on the shop floor. Trajectories of the robot's end-effector are not the most important aspect to be controlled, but rather the initial and final position/orientation of the end-effector. Transfer operations require high positioning accuracy, and high-speed controllers for achieving short transfer times. An ordinary robot controller, which is normally delivered with a commercial robot, can not fulfil these requirements. Therefore, a lot of investigations related to these problems have been conducted.

Improvements of the performance of industrial robots have been achieved in the works by Torfs and De Schutter [77, 78], and Deniard et al. [27], especially in case of joint and/or link flexibilities. Their experimental results show high-performance point-to-point operations of a flexible robot arm, and good tracking and disturbance rejection capabilities. These results indicate great potential improvements; e.g.

in spot welding operations, handling heavy payloads while moving at high speeds, and so on. The problems arise when the payload is beyond the robot capacity. The most logical solution is to utilize a robot with higher capacity or utilize more than one robot manipulator.

An example of a more intelligent operation is the tracking of an unknown contour. This operation is representative for applications like polishing of complex surfaces, automatic glueing of car windows, robotic deburring, etc. Tracking incorporates several control aspects namely position control, force control, and tracking control in order to adapt the orientation of the task frame to the actual tangential and normal direction (fig. 1.1 a).

Usually, this task is performed by using a single robot arm while the contoured object is mounted at a fixed position. This results in a limited attainable space on the object, especially if the contour has a 3-D circular shape. The robot cannot easily reach the object from all directions. This constraint reduces the capabilities of the robot manipulator. A promising solution is to use a multi-robot system to carry out this operation.

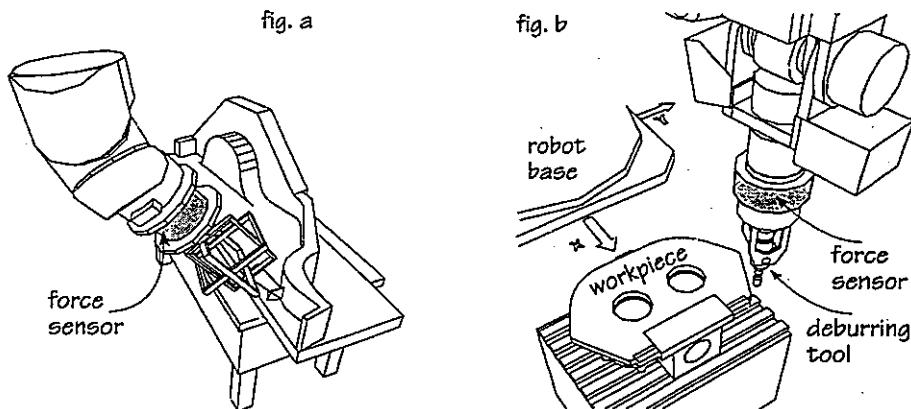


Figure 1.1 : Typical examples of advanced robotic applications. Fig. a is tracking of a 3-D seam as in a welding operation, and Fig. b is deburring of a casting.

Robotic deburring is an example of a complex cutting operation (fig. 1.1 b). The robot controller has to adjust the tangential velocity (feedrate) in the presence of a burr along the edges in order to main-

tain a constant cutting force. The larger the burr size, the lower the tangential velocity. As a result, a constant cutting force is realized. This implies a constant normal force, since it is proportional to the cutting force. To comply with the variety of the burr size, force control can be applied, and some investigations have been conducted in [6, 49, 52, 89].

In many cases the shape of workpiece edges is usually not known in advance; and therefore, the robot controller has to utilize the measurable contact parameters, e.g. contact forces, possibly together with a vision sensor. This information is used to control the tracking along the edges. The performance of this process can largely be improved by using two cooperating robot arms, since the robot holding the tool can easily reach all sides of the workpiece.

The evolution of robotic applications, together with their advantages and limitations outlined above, motivates research on the control of multi robot systems, and becomes the main background of this dissertation. More specifically, a control strategy has to be developed to perform cooperative tasks using a multi robot system. In particular two cooperating robots are studied. The work is based on a long tradition of research and development of compliant robot motion, which has been successfully implemented on some industrial robots at the PMA division of the K.U. Leuven.

Multi-robot systems should be able to cooperate to handle common loads and perform assembly operations, and should be largely autonomous. Consequently, coordination between the robots is essential. Control of cooperating robot arms will add an extra dimension to robotic applications in the future. The success in this research area will certainly much improve robot utilization, increase the productivity and efficiency of the manufacturing processes, and fulfil the requirement for high safety operations in hazardous and dangerous environments.

The advantages of multi robot systems are very useful when manipulating very complex objects, in the sense that the robot is able to reach the manipulated object easily from all directions. It would be very effective, for instance to deburr a complex shaped object from different directions all in one step (fig. 1.2). Tracking of an unknown contour would be easily performed without having to worry about joint limits or singularity problems when a robot is controlled knowing the

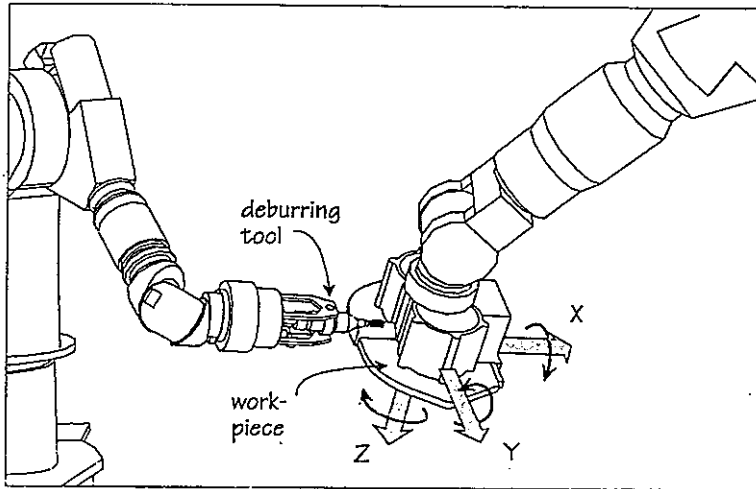


Figure 1.2 : Application of a multi robot system, an example of robotic deburring.

state of the other robot.

The numerous applications of multi robot systems may be classified in two main groups. In the first category, all robot arms are in rigid contact with the manipulated object. The object may or may not be in contact with the environment. Hence, the tasks may be transfer of a large common object or combination of transfer and force exertion by the object on the environment. The second category, involves tasks where each arm holds a separate object, either part and tool, or parts to be assembled. In this work, the second category will mainly be investigated.

The thesis is organized as follows: Chapter 2 gives an overview of the background of the research work, and of the existing robot control techniques for single robot arms as well as for multi robot systems. Chapter 3 describes task specification for cooperating robot arms. This task specification is an extension of the ones introduced by Mason [60] and enhanced by De Schutter [28]. As an extension, a specification is developed to exploit the kinematic redundancy of the system in the robot controller. Chapter 4 treats the useful characteristic of cooperating robot arms as a redundant manipulator. This chapter explains in more detail the physical meaning and the use of the non-square Jacobian matrix for performing cooperative tasks. Chapter 5 describes

the control scheme implemented to control two industrial robots, including the interface to the existing controller *Comrade* [33] and the used programming techniques. Chapter 6 presents simulation and experimental results using two industrial robots the KUKA-361 IR and the KUKA-160 IR. A comparison of different approaches for realizing cooperative tasks is given. A general conclusion summarizes the main results of this work.