Stochastic Models of Election Timing

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Statement of Originality

The work presented in this thesis is the original work of the candidate, except as acknowledged in the text and has not been submitted either in full or in part, for a degree at any other university.

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- Paine, M. *Tip for PM to figure out poll advantage.* The Mercury. February 5, 2004.

Stochastic Models of Election Timing

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Abstract

Under the democratic systems of government instilled in many sovereign states, the party in government maintains a constitutional right to call an early election. While the constitution states that there is a maximum period between elections, early elections are frequently called.

This right to call an early election gives the government a control to maximize its remaining life in power. The optimal control for the government is found by locating an exercise boundary that indicates whether or not a premature election should be called. This problem draws upon the body of literature on optimal stopping problems and stochastic control.

Morgan Poll's two-party-preferred data are used to model the behaviour of the poll process and a mean reverting Stochastic Differential Equation (SDE) is fitted to these data. Parameters of this SDE are estimated using the Maximum Likelihood Estimation (MLE) Method. Analytic analysis of the SDE for the poll process is given and it will be proven that there is a unique solution to the SDE subject to some conditions.

In the first layer, a discrete time model is developed by considering a binary control for the government, viz. calling an early election or not. A comparison between a three-year and a four-year maximum term is also given. A condition when the early exercise option is removed, which leads to a fixed term government such as in the USA is also considered. In the next layer, the possibility for the government to use some control tools that are termed as 'boosts' to induce shocks to the opinion polls by making timely policy announcements or economic actions is also considered. These actions will improve the government's popularity and will have some impacts upon the early-election exercise boundary. An extension is also given by allowing the government to choose the size of its 'boosts' to maximize its expected remaining life in power.

In the next layer, a continuous time model for this election timing is developed by using a martingale approach and Ito's Lemma which leads to a problem of solving a partial differential equation (PDE) along with some boundary conditions.

Another condition considered is when the government can only call an election and the opposition can apply 'boosts' to raise its popularity or just to pull government's popularity down. The ultimate case analysed is when both the government and the opposition can use 'boosts' and the government still has option to call an early election. In these two cases a game theory approach is employed and results are given in terms of the expected remaining life in power and the probability of calling and using 'boosts' at every time step and at certain level of popularity.

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Contents

1 Introduction 1

2 Background 7

2.1 Literature Review 7

2.2 Mathematical Preliminaries 11

2.2.1 Basic Probability Theory 11

2.2.2 Stochastic Process and Brownian Motion 14

2.2.3 Martingales 17

2.2.4 Stochastic Differential Equations 20

2.2.5 The Ito Integral and Ito's Formula 25

2.2.6 Dynamic Programming 28

2.2.7 Game Theory 30

2.2.8 Maximum Likelihood Estimation 33

2.2.9 Numerical Methods for Ordinary Differential Equations 35

2.2.10 Finite Difference Methods 37

2.3 Summary 44

3 The Underlying Process 47

3.1 Transition Probabilities and Parameter Estimation 47

3.2 Winning Probabilities 49

3.3 Sampling and Response Errors 52

3.4 Tests of Assumptions 53

3.4.1 Normality of Increments of S 53

3.4.2 Volatility Estimates 55

3.4.3 Parameter Accuracy 56

3.4.4 Term Structured Volatility Model 57

3.5 Mean Reverting SDE 58

3.5.1 The Deterministic Model 59
3.5.2 Existence and Uniqueness 63
3.5.3 A Class of Polynomial Functions for the Drift 68
3.6 Summary 70

4 A Discrete Time Model for Election Timing (I) 71
4.1 Introduction 71
4.2 Formulation and Solution Method 74
4.3 Results 78
4.3.1 Expected Remaining Life 78
4.3.2 Exercise Boundary 79
4.3.3 No Early Election Permitted 81
4.3.4 Nonconstant Volatility 83
4.4 Conclusions and Further Research 84

5 A Discrete Time Model for Election Timing (II) 87
5.1 Introduction 87
5.2 The Model 88
5.2.1 Notation and Formulation 88
5.2.2 Transition Probabilities and Parameter Fitting 90
5.3 Results 91
5.3.1 Expected Remaining Life 93
5.3.2 Exercise Boundaries 93
5.4 Extension 94
5.5 Conclusions and Further Research 98

6 A Continuous Time Model for Election Timing 101
6.1 Introduction 101
6.2 The Model 101
6.3 Probability of Winning the Election 103
6.4 Numerical Scheme 104
6.4.1 Finite Difference Scheme 104
6.4.2 Algorithm 105
6.5 Results 106
6.5.1 Expected Remaining Life 106
6.5.2 Exercise Boundary 107

6.6 Conclusions and Further Research . . . 109

7 A Game Theory Approach for Election Timing (I) 111

7.1 Introduction . . . 111
7.2 Problem Formulation and Notations . . . 112
7.3 Game Theory and Stochastic Dynamic Programming . . . 113
7.3.1 Model without Lead Time . . . 113
7.3.2 Model with Lead Time . . . 116
7.4 Case Study . . . 118
7.5 Numerical Results . . . 119
7.5.1 Expected Remaining Life . . . 119
7.5.2 Call Probability . . . 120
7.5.3 Boost Probability . . . 121
7.6 Conclusions and Further Research . . . 123

8 A Game Theory Approach for Election Timing (II) 127

8.1 Introduction . . . 127
8.2 Problem Formulation and Notations . . . 128
8.3 Theory . . . 129
8.3.1 Game Theory and Stochastic Dynamic Programming . . . 129
8.3.2 Dominant Strategy . . . 131
8.3.3 Political Time Frame and Algorithm . . . 132
8.4 Case Study . . . 134
8.5 Numerical Results . . . 136
8.5.1 Expected Remaining Life . . . 136
8.5.2 Call Probability . . . 136
8.5.3 Government Boost Probability . . . 139
8.5.4 Opposition Boost Probability . . . 141
8.6 Conclusions and Further Research . . . 144

9 Conclusions and Further Research 147

9.1 Conclusions . . . 147
9.2 Further Research . . . 149

References 151

A Election Results 1901 - 2004 159

B The Election Timetable 161

List of Figures

1.1 Voting Intentions with Election Dates and Significant Events 1960-2002 3

1.2 Two-Party-Preferred (LNP - ALP) with Significant Events 1993-2005 4

3.1 Simulated Process 49

3.2 Histogram of the Distribution of Seats in 2001 Election 51

3.3 (a) Proportion of Seats Won from Each S (b) Probability of Winning the Election from True State S 52

3.4 Volatility Estimates for Two Party Preferred Data 56

3.5 Histograms for $\hat{\mu}$ and $\hat{\sigma}$ 57

3.6 EWMA and Term Structured Volatility Model 58

3.7 EWMA and Term Structured Volatility Model (Best Fit) 59

4.1 Finite State and Discrete Time Model 73

4.2 Expected Remaining Life (μ= 3.98, $\sigma = 0.28$) with (a) Early Election (b) No Early Election 79

4.3 Impacts of μ or σ on the Expected Remaining Life 80

4.4 Impacts of both μ and σ on the Exercise Boundary 80

4.5 Exercise Boundaries for Calling an Election (a) Three-year Term (b) Four-year Term 81

4.6 Impacts of μ and σ on the Exercise Boundary - Three-year Term . . 82

4.7 Impacts of μ and σ on the Exercise Boundary - Four-year Term . . . 82

4.8 Exercise Boundary and Election Called 1949-2004 83

4.9 Non Constant Volatility (a) Expected Remaining Life (b) Exercise Boundary 84

5.1 Expected Remaining Life with $B_{max} = 5$, $\mu = 4.12$, $\sigma = 0.28$ 92

5.2 Expected Remaining Life with $B_{max} = 10$, $\mu = 4.12$, $\sigma = 0.28$ 92

5.3 Call Exercise Boundaries with (a) $B_{max} = 5$ (b) $B_{max} = 10$ 94

5.4 Boost Exercise Boundaries with (a) $B_{max} = 5$ (b) $B_{max} = 10$ 95
5.5 Call and Boost Exercise Boundaries with $B_{max} = 10$ 95
5.6 Expected Remaining Life with $B_{max} = 10$, μ =4.12, $\sigma = 0.28$ (Extension Model) 95
5.7 (a) Call Exercise Boundaries with $B_{max} = 10$ (b) Boost Intensity with $B_{max} = 10$ 97

6.1 Probability of Winning the Election 103
6.2 Expected Remaining Life (a) With Early Exercise Option (b) Without Early Exercise Option 107
6.3 Call Exercise Boundary for (a) Different Maximum Terms (b) Different $P(W|S)$ 107
6.4 Call Exercise Boundary (a) μ fixed, σ vary (b) σ fixed, μ vary 109

7.1 Expected Remaining Life with $B_{max} = 4$ without Lead Time 120
7.2 Expected Remaining Life with $B_{max} = 4$ with Lead Time 120
7.3 Call Probability for $B_{max} = 4$ without Lead Time 122
7.4 Call Probability for $B_{max} = 4$ with Lead Time 122
7.5 Boost Probability for $B_{max} = 4$ without Lead Time 124
7.6 Boost Probability for $B_{max} = 4$ with Lead Time 124

8.1 Expected Remaining Life with $(B^g, B^o) = (0,0), (4,0), (0,4)$ 135
8.2 Call Probability with $B^g = 0$ 137
8.3 Call Probability with $B^o = 0$ 137
8.4 Call Probability with $B^g = B^o$ 138
8.5 Government Boost Probability with $B^o = 0$ 139
8.6 Government Boost Probability with $B^o = 4$ 140
8.7 Government Boost Probability with $B^g = B^o$ 140
8.8 Opposition Boost Probability with $B^g = 0$ 142
8.9 Opposition Boost Probability with $B^g = 4$ 142
8.10 Opposition Boost Probability with $B^o = B^g$ 143

List of Tables

2.1 Payoff Matrix 31

3.1 Parameter Estimates of Term Structured Volatility Model 58

7.1 Payoff Matrix 115

7.2 Payoff Matrix in the Election Mode 117

8.1 Payoff Matrix 131

8.2 Payoff Matrix in the Election Mode 133

Chapter 1

Introduction

Under the democratic systems of government instilled in many sovereign states, the party in government maintains a constitutional right to call an early election. While the constitution states that there is a maximum period between elections (typically 3 or 4 years), early elections are frequently called. For example, the Australian Constitution and Commonwealth Electoral Act 1918 give the Federal Government the right to call an early election, subject to approval by the monarch's representative (the Governor General). However, the presidential elections in the USA do not possess this property, enforcing a fixed period of four years between elections.

This right to call an early election gives the government a control with which to optimize its objective of remaining in power for as long as possible. In some sense, the party in government has an option, which it can freely exercise. In this thesis, we want to devise the optimal control for the government by locating an exercise boundary, which indicates whether or not a premature election should be called. This problem draws upon the body of literature on optimal stopping problems and stochastic control. The problem can be compared with the determination of early exercise for American options in finance.

In case an election is called at some time t, a mechanism is needed to gauge the likelihood of the government being returned to power. We have chosen to use popular opinion polls to measure the voting intentions of the public. Other factors may include an aggregation of expert opinion or bookmakers' dividends. The sampling frequency of this data is not always regular, meaning that we must be careful with time series techniques and parameter estimation. Historical data is readily available from the Internet and some of them are presented on a fortnightly basis in newspapers.

It is certainly true that opinion polls do not necessarily reflect the outcomes of an election (see [38] for the 1997 case in France). Noncompulsory voting, sampling and response errors and importantly the effect of an exaggerated majority (due to the common practice of regional representation) all impact on the probability of re-election. In a noncompulsory voting system, polled persons may not actually intend to vote. Probabilistic methods based on historical precedents encompass these situations in the models presented in this thesis.

Voting in Australian federal elections is compulsory and follows a Majoritorian Alternative Vote system [84]. Voters register preferences for each candidate, and preferences are iteratively distributed until one party achieves the majority of (referred) preferences. This system is also known as "alternate vote" and is used in the House of Representatives and the lower house in every Australian State Parliament except for the Australian Capital Territory (ACT) Legislative Assembly and the Tasmanian House of Assembly where a variation of the Proportional Representation voting system known as *Hare-Clark* system is used ([6]). We concern ourselves with rules for the Australian Federal House of Representatives and use the associated poll and historical electoral data.

The Commonwealth Electoral Act 1918 gives the timeline for each step in holding the election. It begins with the issue of the writs and ends with the return of the writs after the votes have been counted. A writ is a document commanding an electoral officer to hold an election. These steps begin after the expiry or dissolution of the House of Representatives and include: the issue of writs, the close of rolls (the list of voters who are eligible to vote at an election), the close of nominations, the declaration of nominations, polling day and the return of writs. The new parliament must meet within 30 days of the day appointed for the return of writs (see Appendix B, [4] or [5] for details).

The models in this thesis assume that opinion polls are driven by random processes. The announcement, distribution and dissemination of news (whether policy announcements or exogenous news items), drive the voting intentions of the public. Figures 1.1 and 1.2 show the voting intentions between the Coalition (Liberal and National Parties (LNP)) and the ALP (Australian Labor Party) and two-party-preferred (LNP and ALP) voting intentions of the Australian public over the last decade or so, along with significant events. The significant events in these figures were taken from [63], [66] and [91]. Some actions by the government seem to affect the polls, for example the policy announcement of tax breaks. Exogenous events similarly have a significant impact on voting intentions, for example the World Trade Centre terrorist attacks.

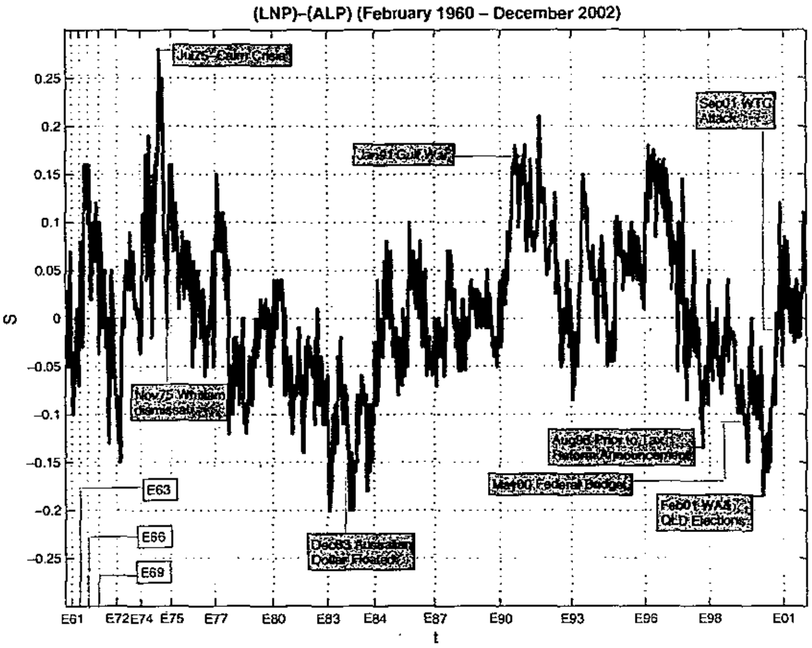

Figure 1.1: Voting Intentions with Election Dates and Significant Events 1960-2002

We gradually build up the models in several layers of complexity. For all of the models, some fundamental assumptions are presumed:

- The government maintains the right to call an election at any time.
- The maximum time between elections is Y years (three years in the Australian House of Representatives).
- A constant lead time T_L is enforced between announcing the election (issuing the writ) and holding the election.
- The democratic electoral system of representative seats means that a party may receive more than 50% of the vote, but still lose the election (exaggerated majority).
- The party system can be described in terms of two main parties.

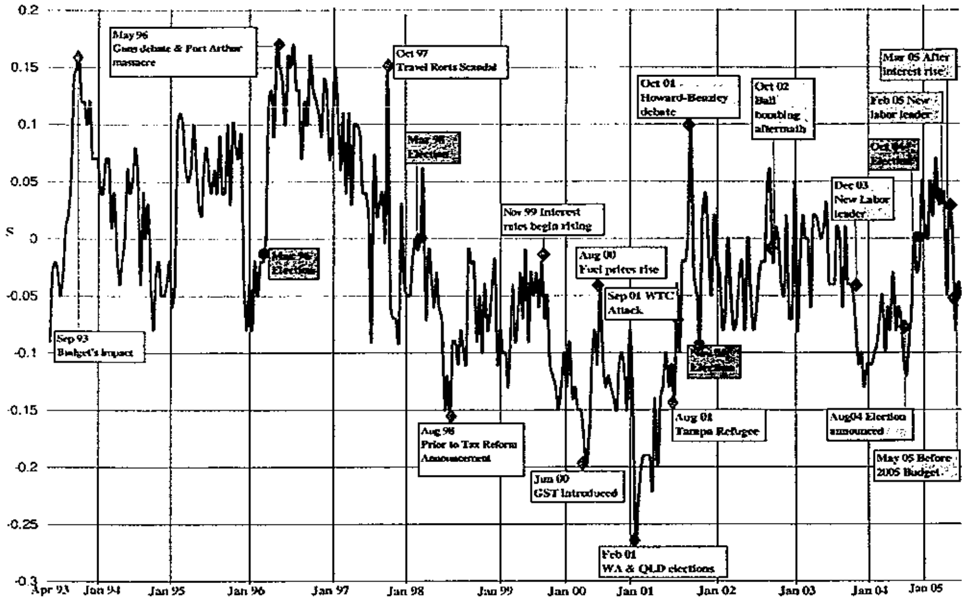

Figure 1.2: Two-Party-Preferred (LNP - ALP) with Significant Events 1993-2005

We model the opinion polls as a mean reverting discrete Markov stochastic process (a finite number of popularity states in discrete time). A recursive formulation for the expected remaining life in power is developed and used to solve the election exercise boundary. The organization of the remainder of this thesis is the following.

In Chapter 2, a literature review about models on election timing that have been developed so far is given. Mathematical preliminaries that are needed to develop the models are also given and include the basic theory of probability, stochastic process and Brownian motion, martingales, the Ito Integral and Ito's Formula. Also, theory on Stochastic Differential Equations (SDEs) is introduced to understand some conditions related to the SDE model for the poll data. Dynamic programming and game theory are also needed in developing algorithms to deal with the models. Finally, the Maximum Likelihood Estimation (MLE) method is introduced to estimate parameters in the mean reverting SDE and numerical methods in solving SDEs are also presented.

The underlying process which consists of transition and winning probabilities, parameter estimation, sampling and response errors is described in Chapter 3. These terms will be used in developing the models in the subsequent chapters. A term structured volatility model is also introduced for the volatility coefficient in the

SDE model. The mean reverting SDE used in the models and its properties are analysed in terms of the existence and uniqueness of the solution.

In Chapter 4, the first finite state and discrete time model of election timing is discussed. In that model, the only option owned by the government is to call or not to call an election. With this model, at every time step at any level of popularity the government must decide whether to call an election or not by considering the maximum expected remaining life in power between calling and not calling an election. A comparison between the expected remaining life for a maximum term of three and four years is given along with the exercise boundary for both cases. The same problem is also considered by using a term structured volatility model and a situation where the early exercise option is removed, which represents the condition in countries with fixed period between elections is also investigated.

In Chapter 5, an extension of the previous model is developed by considering the possibility for the government to use control tools termed 'boosts' to raise its popularity in the polls in addition to its option to call an early election. These control tools include economic policy announcements such as tax cuts or budgets. However, in this chapter it is still assumed that the opposition can do nothing. It is assumed that the government can only apply a boost of magnitude one at a time. Later in that chapter another situation is accommodated, that is the possibility for the government to choose the size of its boost between zero and one to maximize the expected remaining life in power by considering options to call an election and/or to use its boosts.

A continuous time model for election timing is discussed in Chapter 6. Starting with a mean reverting SDE to describe the poll process, a martingale approach and Ito's Formula are used to derive a partial differential equation (PDE) with some boundary conditions. The expected remaining life and the exercise boundary are found by solving the PDE numerically using a Crank-Nicolson method. In this model, the government only has an option to call an election or not. Impacts on the expected remaining life and exercise boundary in relation to a three-year and a four-year maximum term, different values of the parameters of SDE and different functions for the probability of winning the election are also investigated.

In Chapters 7 and 8, a game theory approach is employed to model the election timing. The election timing is considered as a zero-sum game between the government and the opposition in terms of the expected remaining life in power. In Chapter 7, a situation where the government can only call an election while the opposition has a set of its policies to be delivered to the public that will pull the government's popularity down is considered. At every time step at any level of popularity the government should decide whether to call an election or not while the opposition can choose whether to apply its boosts or not. This game theory approach can be represented by a 2×2 payoff matrix. In this approach, results are given in terms of the expected remaining life in power and probabilities for the government and/or the opposition to apply their optimal strategies.

Chapter 8 is an extension of Chapter 7 in which the government can call an election and use its boosts to raise its popularity while the opposition can also apply its boost to pull the government's popularity down. It is assumed that when the government and the opposition use their boosts at the same time, nothing happens in the polls. However, when only one party uses its boosts, the poll will move (up or down) in favour of whoever applies the boost.

Chapter 9 contains conclusions and some directions for future research.