Chapter 9

Conclusions and Further Research

9.1 Conclusions

Several mathematical models for election timing have been developed along with their applications to Australian Federal Elections for the House of Representatives. The models are calibrated against the Morgan Poll data that measure the popularity of the government and the opposition over time. The variable of interest in the models was S, the difference in the popularity from the two-party-preferred data from Morgan Poll. From that data, we fitted a mean reverting SDE to describe the polls process.

Results in most of the models were given in terms of the expected remaining life in power, exercises boundaries (call and/or boost) for the government and the opposition. These exercise boundaries gave an indication for the government and/or the opposition to call an election and/or apply a boost given a certain level of S. The area above this boundary was the exercise region where the government or the opposition should exercise their options. In general, we found that the expected remaining life and exercise boundaries are monotone in time. As time elapses, the expected remaining life decreases especially for low levels of S as the election date is getting closer. In terms of the call exercise boundary, the government needs higher popularity if it wants to call an election earlier. In the models using a game theory approach, results were given in terms of mixed or pure strategies. A pure strategy means that the government and/or the opposition will choose their optimal strategy with probability one while in mixed strategies these probabilities lie between zero and one.

We started with a discrete model for election timing in Chapter 4 where the only control owned by the government is to call or not to call an early election. The opposition on the other hand does nothing. In this model, at every time step the government's decision is maximizing the expected remaining life in power by considering two options: calling or not calling an election. A comparison of the expected remaining life for a three-year maximum term and a four-year maximum term was given and it was found that the expected remaining life is longer in a four-year term. This is quite clear since in a four-year maximum term, the government has more freedom to time the election. A condition where the option of an early election is removed was also considered. This condition covers presidential elections in the USA where the election is held once every fixed period (four years). In this case, the expected remaining life was shorter compared with a situation with an early election option. We also proved that it is best for the government to choose the minimum lead time allowable. A term structured volatility model was employed to describe the dynamic of the poll process in addition to the maximum likelihood estimation for the volatility coefficient in the SDE of the poll process. Some sensitivity analysis of the values of μ and σ on the expected remaining life and the exercise boundary were also given.

An extension to the first model was discussed in Chapter 5 where the option for the government to use so-called "boosts" to raise their popularity in the polls was added. These boosts come in the form of policy announcements or economic actions that can please public and therefore make its popularity a little bit higher. In this model it was still assumed that the opposition can do nothing. In the beginning of its term the government is provided with a discrete number of boosts to be used during its term and at every time step it is assumed that the government can only use a single boost. These boosts will be renewed after the next election and the magnitude of a boost is one. It was found that these "boosts" will raise the expected remaining life in power compared with the results from Chapter 4 as each boost will make the government's popularity a bit higher and thus impact on the expected remaining life. An extension of this model was considered by allowing the government to choose the size of its boosts, $0 \leq \gamma \leq 1$. There were also assumptions that the boosts resources grew linearly with time at a certain rate and the government can carry its boosts across the election period into the next term.

In Chapter 6, a continuous version of the election timing problem was considered. Starting with the mean reverting SDE for the polls process and using a martingale approach and the Ito Lemma, a PDE for the expected remaining life in power was derived along with some boundary conditions. The PDE was solved numerically using a Crank-Nicolson method and results were given in terms of expected remaining life in power and exercise boundaries. A comparison between three-year and four-year maximum terms is also given along with sensitivity analysis of the exercise boundaries in response to changes in parameters μ and σ, different probabilities of winning the election and different maximum terms.

A game theory approach was employed to derive the models in Chapters 7 and 8. In Chapter 7, the government can only call an election while the opposition is provided with "negative boosts" to use to pull the government's popularity down in the polls. In practice, these boosts can be a set of policies which will please the public and therefore will raise its own popularity. This election timing problem was modelled as a zero-sum game between the government and the opposition in terms of the expected remaining life in power. At any time and at any level of popularity the government's strategy is either to call an election or not while the opposition's strategy is either use its boosts or not. In this case we ended up with a 2×2 payoff matrix for the expected remaining life in power. As expected, it was found that the expected remaining life in power depends on the number of boosts available to the government and the opposition. When the government has more boosts, the expected remaining life is longer and the reversed condition occurs when the opposition has more boosts. Also, the opposition should maintain enough boosts to be applied during the election mode to give a maximum impact before the election date. In Chapter 8, the government maintains the right to call an early election and both the government and the opposition are provided with certain number of discrete 'boosts' to be used in order to raise their popularity in the poll. In this situation we dealt with a 2×4 payoff matrix and a concept of dominant strategy is introduced in order to reduce the size of the payoff matrix to 2×2. In both chapters, results were given in terms of the expected remaining life in power, boost and call probabilities. It was found that in general, boosts should be employed at every time step in the election period.

9.2 Further Research

In this thesis, it was assumed that the jump process in the poll reflected by applying 'boosts' was deterministic. For an extension, it would be interesting to consider a jump process with random magnitude and analyse its impacts on the exercise boundary and the expected remaining life in government. In conjuction with this, is to assume that boost resources can grow linearly with time at a certain rate, rather than being refreshed at the beginning of each electoral term.

The application of the models in this thesis was only to the Australian Federal Elections although the models have capabilities to be applied to State Elections or even in other countries with maximum terms election such as UK, New Zealand, Japan or Canada. Application to countries with fixed terms election such as USA using these models is also possible. This would be another possibility for further research.

In the game theory approach, it was assumed that there were only two major parties dominating the election process and labelled as the government and the opposition. In some countries there are more than two major parties dominating the election process and accommodating this situation in the game theory framework would lead to a game theory problem with n players rather than just two players as described in this thesis. This is also a possible direction and a challenging task to pursue for further research.

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