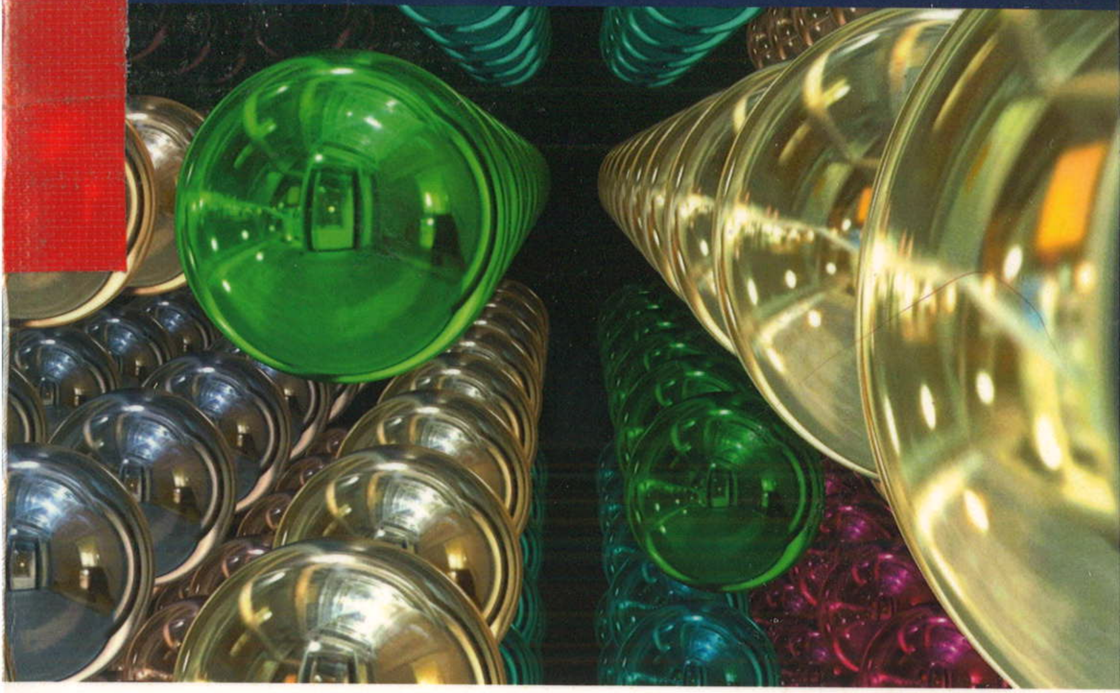

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and Its Applications to Compton Scattering and Hydrogenic Atoms

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CHAPTER 1

Introduction

The theory of quantum mechanics was established in the beginning of the twentieth century. This theory explains the behaviour of physical systems, from elementary particle and atoms, to molecules and solids. One of the basic features of quantum physics is the discrete system which is characterized by a new constant of nature, i.e. Planck's constant h. Bohr in 1913 was able to calculate the discrete energy levels of an atom using the Planck constant and classical canonical variables. The concept of the wave-particle nature of light and especially the introduction of the photon idea was developed by Einstein in order to explain the photoelectric effect. De Broglie in 1923 had also discovered that the duality principle is not only for photons, but for all particles. The modern description of quantum mechanics was established independently by Schrödinger through his famous Hamiltonian eigenvalue problem approach, and by Heisenberg through his matrix mechanical approach. Both approaches were later discovered to be equivalent.

The special theory of relativity was discovered in 1905, several years before quantum mechanics was established. This theory revolutionizes our concept of spacetime, particularly when the motion of an object approaches the speed of light c. Soon after the establishment of quantum mechanics, there were several attempts to develop a 'relativistic' quantum mechanics, which incorporated quantum mechanics with the special theory of relativity. The first published relativistic wave equation, now known as the Klein-Gordon equaton, is

$$(D^\mu D_\mu + \frac{m^2c^2}{\hbar^2})\Psi_{KG0}(x) = 0 \tag{1.1}$$

hereafter referred to as the KG_0 equation. This equation was immediately considered to suffer from several defects. The zeroth component of its conserved current, which in nonrelativistic quantum mechanics represents a probability density is no longer positive definite. Also the fact that the KG_0 equation is a second order equation in time means that the equation is no longer in Hamiltonian form.

Dirac suggested his now famous equation which is a linear equation in time and is related to the free KG_0 equation. It gives a positive definite zeroth component of the associated conserved current, and is a four-component equation in Hamiltonian form. In addition, the Dirac equation describes the existence of particle spin. Since then the Dirac equation has been widely accepted and used to describe spin-$\frac{1}{2}$ particles, although there were some puzzles arising from the equation, such as negative free particle energies and the use of a four-component equation instead of a two-component one which could not be answered at that time.

The most recent attempt to construct another relativistic wave equation for spin-$\frac{1}{2}$ particles was made by Robson and Staudte in 1996 [7, 8], who introduced an eight-component (8-C) relativistic wave equation. It has been shown that this 8-C equation gives the same bound-state energy eigenvalue spectra for hydrogenic atoms as the Dirac equation but that it is associated with different eigenstates due to the different Hamiltonians.

The purpose of this work is to consider the validity of the 8-C relativistic wave equation in the case of two physical problems. The first problem is the scattering of a photon by a free electron : the Compton scattering problem, because it was also the problem used to test the Dirac equation at an early stage. The second problem is the fine structure of the Lyman and Balmer α-lines for hydrogenic atoms. This problem is sensitive to the hydrogenic atomic eigenstates. The results given by the 8-C relativistic wave equation help us to answer the question of how good this equation is compared to the Dirac equation, and may either encourage or discourage us to employ the 8-C equation for future physical applications. As Stephen Hawking has said in his book [1], that each time new experiments are observed to agree with a theory, our confidence in the theory increases, but when a new observation is found to be in disagreement with the theory, we have to either abandon or modify it.

The book consists of six chapters. Chapter 1 is an introduction to this work. Chapter 2 involves a brief review of relativistic quantum mechanics, including the Klein-Gordon theory and the Feshbach-Villars formalism for both a free particle and a particle with an external electromagnetic interaction. Also in this chapter we introduce the operator, expectation value, free particle solution and indefinite metric formalism for both spin-0 theories.

Chapter 3 discusses the relativistic wave equation for a spin-$\frac{1}{2}$ particle which is known as the $FV_{1/2}$ equation. We shall discuss briefly the motivation for deriving this equation. We shall discuss a second order wave equation derived from the Dirac equation, namely the $KG_{1/2}$ equation. From this $KG_{1/2}$ equation, we give the derivation of the $FV_{1/2}$ equation using two representations. We close this chapter by discussing the properties of this theory such as the free particle solutions and the indefinite metric formalism.

Chapter 4 focuses on the uses of the $FV_{1/2}$ equation for Compton scattering. In this chapter we also include a discussion of Compton scattering using the Dirac and $KG_{1/2}$ equations.

Chapter 5 presents a study of the transition probabilities for the components of both the Balmer and Lyman α -lines of hydrogenic atoms using the $FV_{1/2}$ equation. We compare these results with those obtained using the Schrödinger and Dirac equations.

Finally, in Chapter 6, some conclusions and possible future work are given.