



Dharma Lesmono

# Election Timing

Concepts, Mathematical Models and Applications



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Election Timing

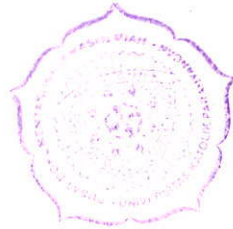
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**To my wife, Sylvia and my children, Rachel and Rafael**

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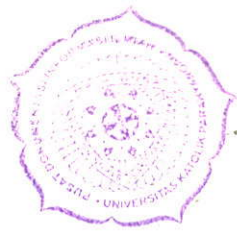
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Bandung, February 2010

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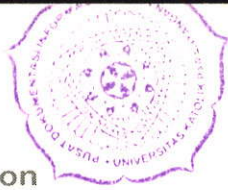
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## Chapter 1 Introduction

Under the democratic systems of government instilled in many sovereign states, the party in government maintains a constitutional right to call an early election. While the constitution states that there is a maximum period between elections (typically 3 or 4 years), early elections are frequently called. For example, the Australian Constitution and Commonwealth Electoral Act 1918 give the Federal Government the right to call an early election, subject to approval by the monarch's representative (the Governor General). However, the presidential elections in the USA do not possess this property, enforcing a fixed period of four years between elections.

This right to call an early election gives the government a control with which to optimize its objective of remaining in power for as long as possible. In some sense, the party in government has an option, which it can freely exercise. In this monograph, we want to devise the optimal control for the government by locating an exercise boundary, which indicates whether or not a premature election should be called. This problem draws upon the body of literature on optimal stopping problems and stochastic control. The problem can be compared with the determination of early exercise for American options in finance.

In case an election is called at some time  $t$ , a mechanism is needed to gauge the likelihood of the government being returned to power. We have chosen to use popular opinion polls to measure the voting intentions of the public. Other factors may include an aggregation of expert opinion or bookmakers' dividends. The sampling frequency of this data is not always regular, meaning that we must be careful with time series techniques and parameter estimation. Historical data is readily available from the Internet and some of them are presented on a fortnightly basis in newspapers.

It is certainly true that opinion polls do not necessarily reflect the outcomes of an election (see [39] for the 1997 case in France). Noncompulsory voting, sampling and response errors and importantly the effect of an exaggerated majority (due to the common practice of regional representation) all impact on the probability of re-election. In a noncompulsory voting system, polled persons may not actually intend to vote. Probabilistic methods based on historical precedents encompass these situations in the models presented in this monograph.

Voting in Australian federal elections is compulsory and follows a Majoritarian Alternative Vote system [89]. Voters register preferences for each candidate, and preferences are iteratively distributed until one party achieves the majority of (referred) preferences. This system is also known as "alternate vote" and is used in the House of Representatives and the lower house in every Australian State Parliament except for the Australian Capital Territory (ACT) Legislative Assembly and the Tasmanian House of Assembly where a variation of the Proportional Representation voting system known as Hare-Clark system is used ([6]). We concern ourselves with rules for the Australian Federal House of Representatives and use the associated poll and historical electoral data.

The Commonwealth Electoral Act 1918 gives the timeline for each step in holding the election. It begins with the issue of the writs and ends with the return of the writs after the votes have been counted. A writ is a document commanding an electoral officer to hold an election. These steps begin after the expiry or dissolution of the House of Representatives and include: the issue of writs, the close of rolls (the list of voters who are eligible to vote at an election), the close of nominations, the declaration of nominations, polling day and the return of writs. The new parliament must meet within 30 days of the day appointed for the return of writs (see Appendix B, [4] or [5] for details).

The models in this monograph assume that opinion polls are driven by random processes. The announcement, distribution and dissemination of news (whether policy announcements or exogenous news items), drive the voting intentions of the public. Figures 1.1 and 1.2 show the voting intentions between the Coalition (Liberal and National Parties (LNP)) and the ALP (Australian Labor Party) and two-party-preferred (LNP and ALP) voting intentions of the Australian public over the last decade or so, along with significant events. The significant events in these figures were taken from [68], [71] and [96]. Some actions by the government seem to affect the polls, for example the policy announcement of tax breaks. Exogenous events similarly have a significant impact on voting intentions, for example the World Trade Centre terrorist attacks.

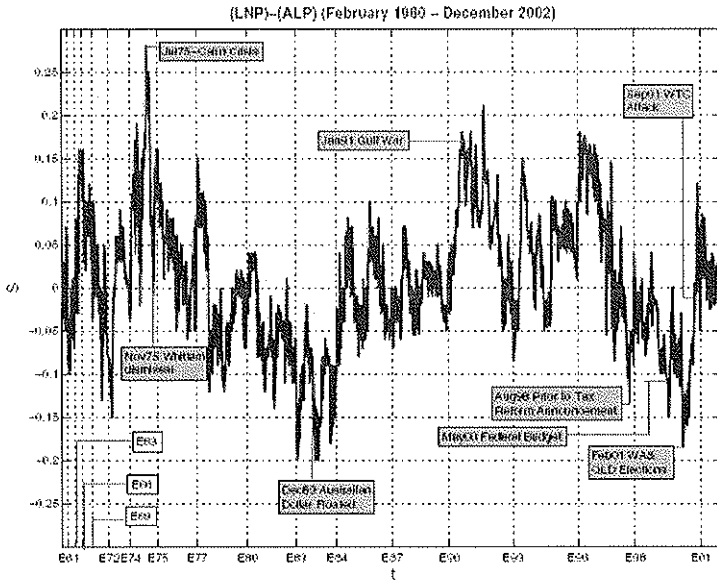


Figure 1.1. Voting Intentions with Election Dates and Significant Events 1960-2002

We gradually build up the models in several layers of complexity. For all of the models, some fundamental assumptions are presumed:

- The government maintains the right to call an election at any time.
- The maximum time between elections is  $Y$  years (three years in the Australian House of Representatives).
- A constant lead time  $T_L$  is enforced between announcing the election (issuing the writ) and holding the election.
- The democratic electoral system of representative seats means that a party may receive more than 50% of the vote, but still lose the election (exaggerated majority).
- The party system can be described in terms of two main parties.

We model the opinion polls as a mean reverting discrete Markov stochastic process (a finite number of popularity states in discrete time). A recursive formulation for the expected remaining life in power is developed and used to solve the election exercise boundary. The organization of the remainder of this monograph is the following.



In Chapter 4, the first finite state and discrete time model of election timing is discussed. In that model, the only option owned by the government is to call or not to call an election. With this model, at every time step at any level of popularity the government must decide whether to call an election or not by considering the maximum expected remaining life in power between calling and not calling an election. A comparison between the expected remaining life for a maximum term of three and four years is given along with the exercise boundary for both cases. The same problem is also considered by using a term structured volatility model and a situation where the early exercise option is removed, which represents the condition in countries with fixed period between elections is also investigated.

In Chapter 5, an extension of the previous model is developed by considering the possibility for the government to use control tools termed 'boosts' to raise its popularity in the polls in addition to its option to call an early election. These control tools include economic policy announcements such as tax cuts or budgets. However, in this chapter it is still assumed that the opposition can do nothing. It is assumed that the government can only apply a boost of magnitude one at a time. Later in that chapter another situation is accommodated, that is the possibility for the government to choose the size of its boost between zero and one to maximize the expected remaining life in power by considering options to call an election and/or to use its boosts.

A continuous time model for election timing is discussed in Chapter 6. Starting with a mean reverting SDE to describe the poll process, a martingale approach and Ito's Formula are used to derive a partial differential equation (PDE) with some boundary conditions. The expected remaining life and the exercise boundary are found by solving the PDE numerically using a Crank-Nicolson method. In this model, the government only has an option to call an election or not. Impacts on the expected remaining life and exercise boundary in relation to a three-year and a four-year maximum term, different values of the parameters of SDE and different functions for the probability of winning the election are also investigated.

In Chapters 7 and 8, a game theory approach is employed to model the election timing. The election timing is considered as a zero-sum game between the government and the opposition in terms of the expected remaining life in power. In Chapter 7, a situation where the government can only call an election while the opposition has a set of its policies to be delivered to the public that will pull the government's popularity down is considered. At every time step at any level of popularity the government should