

# Seismic $P$ - $\delta$ effects in medium height moment resisting steel frames

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(Received August 1990; revised January 1991)*

An analytical investigation of the  $P$ - $\Delta$  effects in medium height steel moment resisting frames to selected earthquake motions is reported. The frames with various design drifts and fundamental natural periods were designed according to the load provisions for New Zealand seismic risk zone A of the Draft Code of Practice for General Structural Design and Design Loading for Buildings, DZ4203. The effect of strength degradation was investigated. Dynamic magnification factors for the member forces and displacement are also presented. Limits for the maximum plastic hinge rotation, inelastic drift, curvature ductility, displacement ductility and prediction of maximum plastic hinge rotation based on the inter-storey drift are suggested.

**Keywords:** steel frames, earthquake motions

Many previous investigations of  $P$ - $\Delta$  effects have indicated that the combination of large gravity loads and lateral displacement, especially in medium to high rise buildings, could cause this second-order effect to become significant. In these investigations the level of significance of the  $P$ - $\Delta$  effect has been expressed in terms such as stability indices, drift indices and the ratio of base shear to total mass. When the  $P$ - $\Delta$  effect becomes significant, consideration must be given to the large increases in displacement, curvature ductility demand, plastic hinge rotation and drift in order to maintain the stability and serviceability of the structures.

There are many different approaches to solving for  $P$ - $\Delta$  effects. In static analyses the increase of secondary moment can be taken into account as the product of relative inter-storey displacement and the vertical force, or an incremental analysis with updated coordinates can be carried out. In dynamic time-history analyses the effect of changing coordinates must be taken into account in every step of the time-history analysis.

For first-order static load analyses using a drift limit and stability index at a certain level of loading as in a previous study<sup>1</sup>, the effect of  $P$ - $\Delta$  can be dealt with in a simple, practical way. The complexity of  $P$ - $\Delta$  effects in dynamic time-history analyses arise because of the characteristics and the intensities of different earthquakes and structural properties of the materials.

In this study, 13 two-bay steel moment resisting frames were designed for various 'design drifts' (see the later section on seismic design and analysis of frames) and fundamental periods, based on the DZ4203

equivalent static approach with a basic seismic acceleration coefficient for seismic zone A<sup>2</sup>. The behaviour of the frames has been investigated by inelastic time-history analyses using five different earthquake records.

Relationships between the results of the equivalent static analysis and dynamic time-history analysis were drawn to convert the inelastic time-history analysis to the static analysis based on DZ4203 since dynamic time-history analyses are expensive to carry out. Design drift limits are recommended and the influence of drift limitations on plastic hinge rotation, curvature ductility and on designing the frame lateral stiffness are also described.

## Previous studies relation to $P$ - $\Delta$ effects

Paulay<sup>3</sup> discussed the probable effect of  $P$ - $\Delta$  moments on inelastic dynamic frame response for reinforced concrete structures. It was suggested that if the strength demand due to the  $P$ - $\Delta$  effect exceeds 15% of the ideal lateral load carrying capacity of a sub-frame, the strength demand should be increased. The  $P$ - $\Delta$  effect should be considered by evaluating the stability index from the expression

$$Q_r = W_{rr} \cdot \delta / (\Sigma M_i - W_{rr} \cdot \delta)$$

By expressing the storey drift,  $\delta$ , in terms of the average slope of the frame (i.e.  $\Delta_u/H$ ), the average of the storey heights above and below the floor under consideration, and a suitable displacement magnification factor  $\lambda$  which relates the storey drift to the average slope of the frame,